Chapter 6: Speed of Light

6.1 Introduction

The speed of light is one of the most fundamental constants in physics. It ties the dimension of time to Euclidean 3-space to form a four-dimensional entity called spacetime. In ordinary three dimensional Euclidean space, there is a familiar invariant, the distance between two points given by:

$$\Delta s^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 \tag{6.1}$$

The value of Δs is independent of the orientation of coordinate axes, *x, y* and *z*. Any rotation transformation on these coordinates must maintain the same value for the Pythagorean distance.

In spacetime, the equivalent invariant is:

$$\Delta s^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 + c^2 \Delta t^2 \tag{6.2}$$

This extended definition describes an invariant not only under rotations of *x, y* and *z* but also including transformations that relate inertial reference frames with constant relative velocities. In our everyday experience, relativistic effects are almost unobservable simply because the speed of light is so high, c =299,792,458 meters/second. This means that the time delays that can be measured in a classroom are too small to be detected by our biological senses. The point of this experiment is to circumvent this problem by using fast electronics to record time differences that otherwise would be hard to measure. The speed of light is large but it is not infinite.

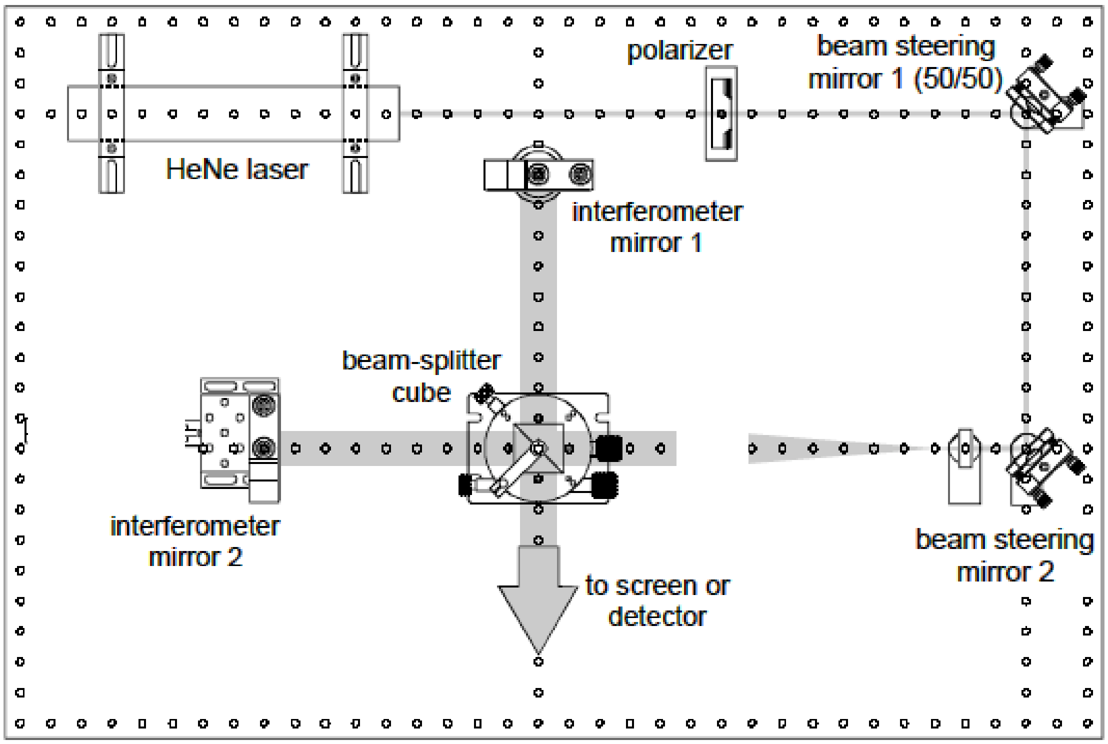
As a prelude to this experiment, you will first become familiar with an oscilloscope and the handling of short electrical pulses. This provides nice examples of one-dimensional wave behavior that are otherwise only easy to exhibit with mechanical models.

6.2 Michelson Interferometer

The Michelson interferometer is one of the most important instruments in optical technology. Michelson used this device to search for a preferred direction for the speed of light. The failure of this search is the cornerstone of Einstein's theory of special relativity. The two largest examples of this instrument are located in Richland, Washington and Livingston, Louisiana. These devices have perpendicular arms each 4 kilometers long and an optical system designed to detect fringe shifts much smaller than 10^{-3} of a wavelength. This ambitious project is attempting to find supernova explosions, gamma-ray bursts and other exotic astrophysical phenomena that generate gravity waves that can cause tiny changes in the length of one arm relative to the other. (Web site: http://www.ligo.caltech.edu).

Interferometers are also interesting devices because they display the phenomenon of coherence central to quantum mechanics. This instrument only functions when light can travel down both arms simultaneously and we have no way of knowing which path a photon actually followed.

The Michelson interferometer is an exquisitely sensitive instrument. As such, it may be frustrating to use for the first time. You may leave this lab with a great respect for Michelson who was able to observe shifts in the fringe pattern by 0.01 fringe when the apparatus was rotated 90°, even with the technology of 100 years ago. Our apparatus is similar to the one in the figure. The two beam steering mirrors 1 and 2 serve only to steer the laser beam onto the beam splitter, which is just a half-silvered mirror. About half the light goes to interferometer mirror 1 and half to interferometer mirror 2. The light reflected from each is recombined at the beam



splitter, and when the combined amplitudes are in phase, the interference pattern appears on the screen.

Alignment of the Interferometer

An essential requirement for seeing interference fringes is parallelism of the two recombined beams. A little bit of geometry shows that if fringes are to be separated by a distance, Δx, the angle between beams must be related by:

$$\sin\frac{\theta}{2} = \frac{\lambda}{\Delta x} \tag{6.3}$$

For a fringe spacing of 1 mm, the misalignment angle must be of the order of 0.07°. Larger fringe spacing requires angular precisions proportionally smaller.

First mount the laser on the stand provided. Carefully screw the expander tube into the threads on the front of the laser. The mirrors are delicate front surface mirrors, so never touch their surfaces. The alignment of the steering mirrors is not too critical. They should be screwed down about midway in their adjustment slots with the mirrors about 45° to the beam. Align the first so that the beam from the laser is approximately centered on it and the reflected beam is roughly centered on the second mirror. Putting a pencil point in the beam is a good way to see it. Align the 2nd steering mirror with the thumbscrews so that the beam reflected off it is centered in the aperture of the beam splitter.

Check that the separations between the splitter and the two interferometer mirrors are nearly equal. Then check if the beam is contained within the first interferometer mirror. If not, you may have to slide the second steering mirror in its mounting screws and realign.

Next look at the spot on the 2nd interferometer mirror. Check that the beam spot is contained within it, and the reflected spot is centered on the side aperture of the splitter.

Now look at the 2 beams coming out of the splitter with a paper screen. Adjust the two beams so they exactly coincide with equal intensities.

At this point, look for interference fringes on the screen. Taping a linear polarizer over the splitter aperture may enhance them. Make small changes in the adjustments to get the clearest pattern. If this doesn't work, try removing the expander from the laser and aligning the two small spots carefully on each other, then replacing the expander.

You can ascertain whether the fringes are due to interference or the structure of the light source by blocking the light in either leg. The alternating fringe pattern should go away. Breathing on one light path will cause a fringe shift because the small change in the index of refraction.

You can use this device to measure the wavelength of light fairly directly. The idea is to interpose a 1 mm thick microscope slide in the beam and vary the incidence angle. For normal incidence, the extra optical path length through the glass slide is $(n\text{-}1)d$ where n is the index of refraction and d is the slide thickness. If the incidence angle departs from normal, the path length in glass increases. With some geometry and trigonometry, one can show that the change in path length relative to normal incidence is:

$$\Delta L = 4d\left(\sin^2\frac{\theta}{2} - n\sin^2\frac{\theta'}{2}\right) \tag{6.4}$$

where θ and θ' are the ray angles in the air and glass with respect to the surface normal. For the Michelson interferometer, the change in path length is doubled because the beams pass through twice, once in each direction. Such variation in path length causes the fringes to move correspondingly. A specific spot on the screen will vary in intensity as a function of glass angle as shown in Figure 6.1 for n=1.5, d=1 *mm* and λ=500 *nm*.

Verify this dependence by measuring the angular position of the glass as you move from interference fringe to fringe. Start with the beam at normal incidence to the glass (θ=0°) with the fringe pattern set for the broadest possible fringes. Mark one dark fringe on the screen and record the angle of the stage. Turn the stage until the next fringe moves to the mark and record the angle. Continue this for a number of fringes. Use the Excel spreadsheet provided with n=1.5, the laser wavelength and the thickness of the glass. [Note: θ' is obtained from θ using Snell's law.]

- Compare your results with the Excel spreadsheet predictions; if necessary, adjust the zero angle to get the best agreement.

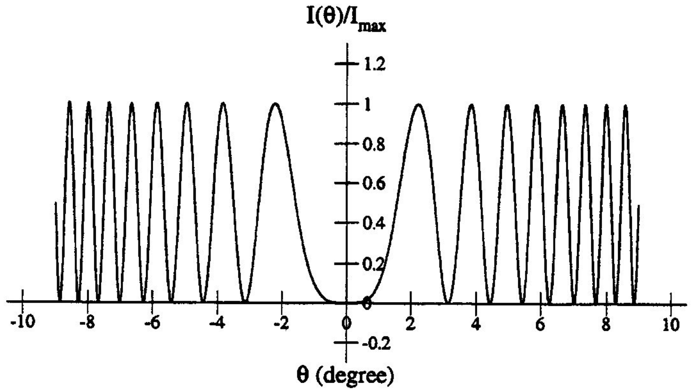
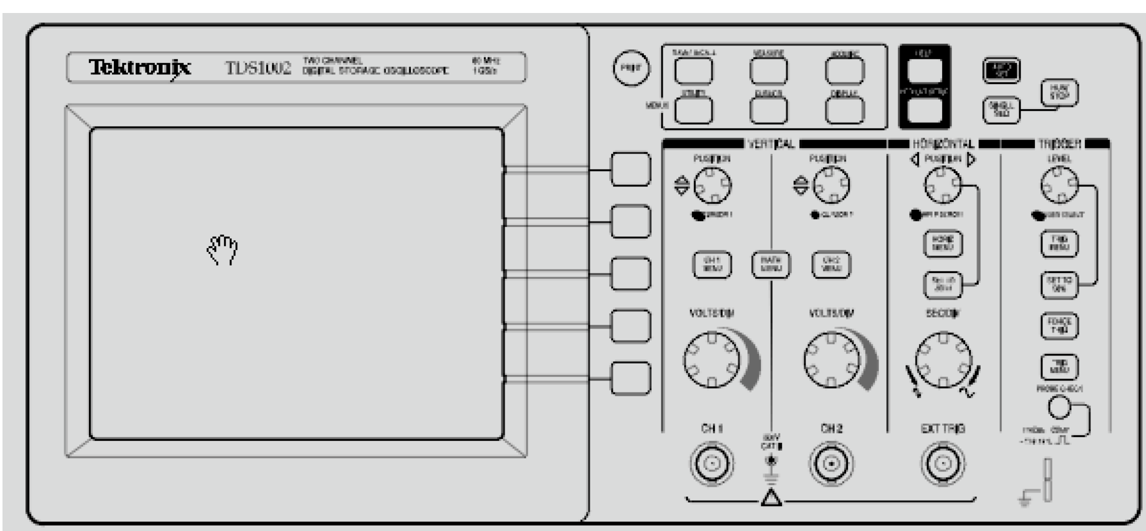


Figure 6.1: Michelson interferometer pattern as a function of angle.

6.3 Oscilloscope

The oscilloscope is an extremely useful instrument for viewing electrical signals. Its most common application is to show how a voltage varies with time. We won't go into the principles of operation here, other than to say it a close relative of a TV set. In this experiment we will be using Tektronix 2002 digital oscilloscopes, mainly for accurate time difference measurements. Most of the settings are made through self-explanatory menus, with instructions appearing on the screen. A "Help" screen is available (though not very convenient).

First spend some time becoming familiar with the operation. The best way to do that is to go to Sect. 6.5.1. With proper setup you should be able to see the pulse from the pulser followed by the reflected pulse ~100 ns later. The time difference can be measured conveniently with the cursors available through the Cursors menu. If you do not obtain a satisfactory scope trace after fiddling with the knobs for a few minutes, ask your instructor for help.



6.4 Pulse Generator

Part of this experiment is devoted to measuring the propagation speed of fast signals in a coaxial cable and determining the behavior of pulses reflected from the ends of such cables. The pulse generator for these observations is a "NIM Pocket Pulser" manufactured by Phillips Scientific. The output pulse is a negative-going signal with an amplitude of about 0.8 volts into a single 50-ohm load. The pulse width is approximately 8 nanoseconds, full width at half maximum (usually abbreviated FWHM). To keep the internal battery from discharging unnecessarily, the circuit only works when connected to an external load with 50Ω or less DC resistance. That means you will need at least one 50Ω coaxial termination somewhere in your circuit, preferably as close to the pulser as possible, to avoid confusing pulse reflections.

6.5 Coaxial Cables

The coaxial cables used in this experiment are standard components for connecting fast pulse circuitry. The cable, called "RG-58C/U", has a characteristic impedance of 50 Ω (TV coax is usually 75 Ω). A scrap of this material is available to you for dissection. It consists of a central conductor inside a polyethylene tube, which is in turn wrapped with a coaxial wire braid return path. The entire assembly is encased in an insulating jacket to provide protection against abrasion and corrosion.

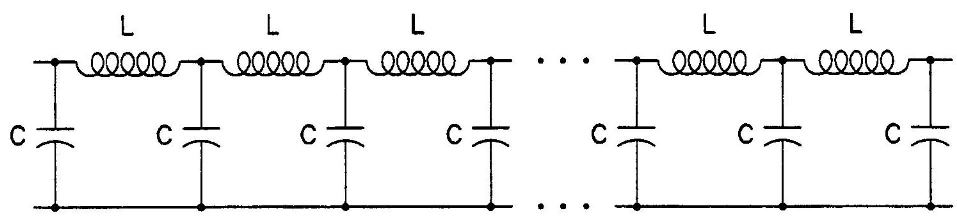


Figure 6.2: Coaxial cable lumped constant equivalent circuit.

A coaxial cable can be modeled electrically as an infinite chain of small inductors and capacitors as shown in Figure 6.2. The characteristic impedance of the cable is given by:

$$Z_0 = \sqrt{\frac{L}{C}} \tag{6.5}$$

where L and C are respectively the inductance and capacitance per unit length. These can be estimated from the formulas for these parameters for cylindrical geometry:

$$\begin{aligned} L &\cong \frac{\mu_0}{2\pi}\left(\frac{1}{2} + \log\left(\frac{r_1}{r_0}\right)\right) \\ C &\cong 2\pi\varepsilon_0 k\left(\log\frac{r_1}{r_0}\right)^{-1} \end{aligned} \tag{6.6}$$

where r_0 and r_1 are the conductor inner and outer radii and k is the dielectric constant of the intervening insulating material (k=2.26 for polyethylene). Thus:

$$Z_0 \cong \frac{\mu_0 c}{2\pi} \sqrt{\left(\frac{1}{2} + \log\frac{r_1}{r_0}\right)\left(\log\frac{r_1}{r_0}\right) / k} \tag{6.7}$$

The basic equation for coaxial cable impedance, Eq. 6.5, can be derived from a simple recursive argument based on Figure 6.2. You are invited to discover how this works.

6.5.1 Signal Reflection

In Physics 340 you learn about one-dimensional waves propagating down taut strings or similar mechanical structures. The same characteristic reflection phenomena occur for electrical pulses in coax cables. There are essentially three limiting cases of interest: If the cable is terminated with a resistance equal to the characteristic cable impedance (50 Ω in this case), any pulses will be totally absorbed and no reflection will occur. If, instead, the cable is shorted, the total electric field must locally vanish and a reflected pulse is generated with opposite polarity (a similar argument was used to describe why reflected circularly polarized light reversed rotation direction). Finally, if the end of the coax cable is left open, the total magnetic field must vanish since the current is zero. This induces a reflected pulse with the same polarity as the original.

Connect the pocket pulser with a “tee” to a 50 Ω terminator and a length of coax cable whose opposite end is attached to the Tektronix scope. At the scope, also include another “tee” followed by a second cable (see Figure 6.3). Trigger the scope on the original pulse from the generator and observe pulse reflections from the end of the second cable under various termination conditions. To make sure what kind of termination you are using, you can check the resistance with a DVM.

- Compare the shapes of the reflected pulses with no termination, 50 ohm, and a short.

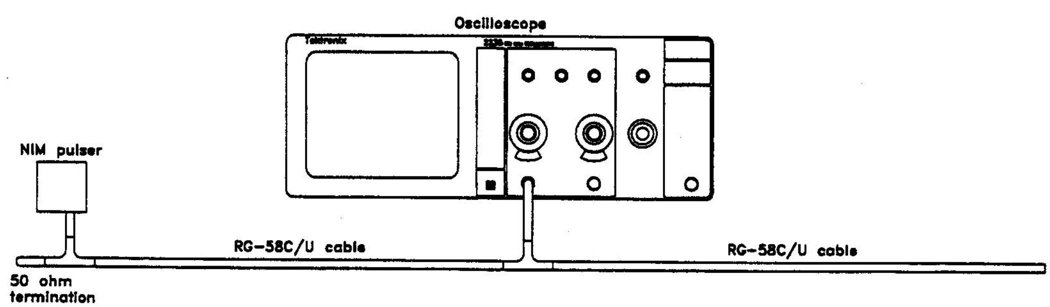


Figure 6.3: Equipment for measuring pulse propagation in coaxial cables.

6.5.2 Speed of Propagation

Now that you can see the effects of unmatched cable terminations, use the reflected signals to measure the propagation speed.

- First, check that doubling the cable length increases the pulse delay proportionally.
- Then measure the length of a much longer cable and use the delay time determined from the oscilloscope to find the propagation speed. Remember that the pulse must go twice the physical length of the cable to get back to the scope input.
- Compare this propagation speed with the speed of light.

6.6 Speed of Light

The speed of light in air can be determined in a manner similar to that in Section 6.5. The apparatus used for this experiment provides a very short pulse to a light emitting diode. The light rays from this source are gathered in a parallel bundle and directed to a corner reflector, which returns the beam along the same path. This light is detected by a photodiode and amplifier and the resulting waveform can be displayed on the oscilloscope. By measuring the variation in time delay with path length, the speed of light can be directly determined. Although the speed of light is quite high, an easily measurable delay time can be observed with the dimensions of the Physics 341 lab.

6.6.1 Measuring Instrument

(The following was copied from the Leybold instruction sheet)

This apparatus, consisting of the light speed measuring instrument and 2 corner reflectors, permits measurement of the speed of light from the path length and the transit time of extremely short light pulses. The light pulses are reflected in a large corner reflector so that they pass through the measurement arrangement twice and are then converted to voltage pulses, which are displayed on the oscilloscope for determination of the transit time.

This configuration uses measuring distances of 10 m to 20 m. To permit measurement of the transit times of approximately 65 ns to 130 ns using standard oscilloscopes, the light speed-measuring instrument is equipped with a trigger output and a quartz-stabilized 10 MHz output. The signals for external triggering of the oscilloscope are optimally matched to the voltage pulses, so that complete pulses can be displayed at all times. Where necessary, the signal of the 10 MHz output can be used on the second oscilloscope channel as a calibrated time base; the edges of the signal (period 100 ns) then serve as a measuring standard that is independent of the properties of the oscilloscope.

6.6.2 Measuring Principle

The apparatus is pictured in Figure 6.4. A high-performance LED (a) driven by an oscillator circuit emits short red light pulses with a repetition frequency of 40 kHz. The beam passes through a semitransparent mirror, which acts as a beam divider (b}; one half is reflected as a "reference beam" (see below) toward housing window (c_0); the other half, the "measuring beam", passes through window (c_1). A lens (f = 200 mm) images the LED on a corner reflector (d_1), which is (by definition) at a distance $s/2$; this mirror reflects the beam precisely into itself and, via beam divider (b), into a receiver diode (e). This converts each light pulse that has traveled the path s into a voltage pulse, which is displayed on the oscilloscope as the "time signal" U_1.

If the reflector is moved by $\Delta s/2$, so that the light path changes by Δs, the time signal U_1 on the oscilloscope is shifted by Δt. The speed of light can be determined from the slope of the line $\Delta s = f\Delta t)$ by recording multiple value pairs.

To measure the absolute time t that a light pulse requires to travel path s, a reference signal U_0 for the zero point is required in addition to the time signal U_1; this reference signal is generated using a second, small reflector (d_2). This is set up either on window (c_0) in the light beam, which is reflected upward by the beam divider (b) or directly in the measuring beam at window (c_1), and reflects the reference beam to receiver diode (e). The two light paths are equivalent.

The reference beam, which is converted into a voltage by the receiver diode, is displayed on the oscilloscope as reference signal U_0. The time difference between U_1 and U_0 is the time t that the light requires to travel path s.

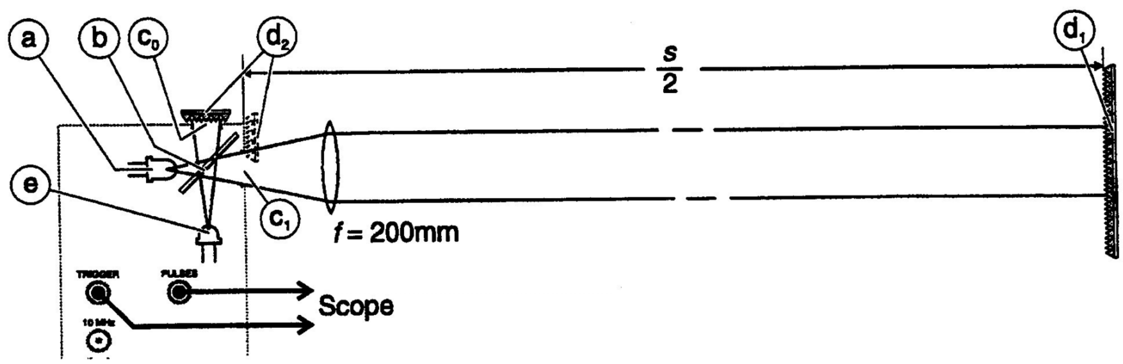


Figure 6.4: Beam path in the experiment setup for determining the speed of light.

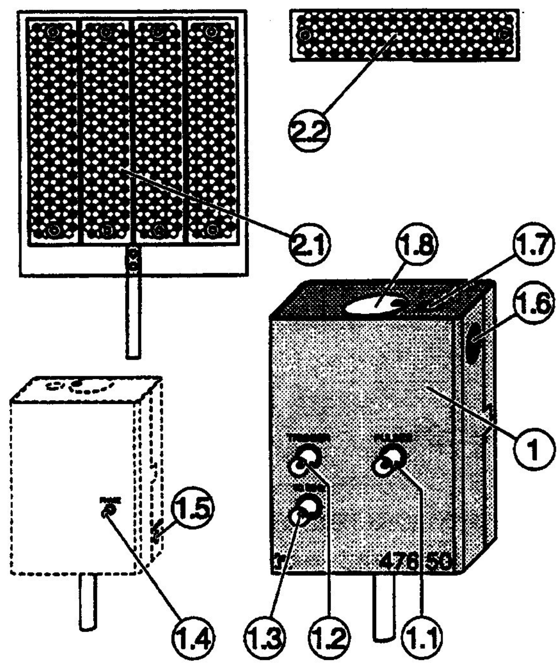


Figure 6.5: Speed of light measuring instrument.

6.6.3 Instrument Description, Technical Data

The light speed measuring instrument as shown in Figure 6.5 comprises the pre-adjusted optical elements, source LED, beam divider, and receiver diode, as well as the electronics for generating the pulses available at outputs (1.1) to (1.3) (output resistance 50Ω).

A detailed description for each knob or output is provided below:

(1.1) Pulse output: Supplies the voltage pulses obtained from the transmitter LED via the receiver diode as time signals for time measurement on the oscilloscope. Pulse characteristics:

- Repetition frequency; 40 kHz
- Peak width: 20 ns
- Half-value peak width: typically 5 ns
- Wavelength of light: 615 nm

(1.2) Trigger output: The pulses trigger the oscilloscope externally. The positive edge of the square-wave trigger voltage (TTL-signal) leads the emission of the light pulses by about 60 ns. As a result, the complete measuring signals can be displayed on standard oscilloscopes even for extremely short light paths (*e.g.,* reflector directly in front of windows (1.6) or (1.7)).

(1.3) 10 MHz output: The quartz-stabilized AC voltage signals with a period of 100 ns can be used on the second oscilloscope channel as a calibrated time base. The edges of the signal serve as a measurement standard:

- when the time measurement in the calibrated ranges of the oscilloscope cannot be carried out under optimum use of the oscilloscope screen or
- when the time base of the oscilloscope is to be checked and/or corrected.

The signal shape depends on the bandwidth of the oscilloscope used. For a 100 MHz instrument the edges are steeper, the signals resemble a square-wave.

(1.4) Phase adjuster: Permits a phase-shift of the 10 MHz signal over 1 period for matching to the phase relation of the measuring signal (optimizing the evaluation conditions, see Section 6.6.7).

(1.5) Power input: Socket for connecting the plug-in power supply with Cannon plug 12 V, 20 W (562 791 for 230 V AC or 562 792 for 115 V DC)

(1.6) Window for measuring beam

(1.7) Window for reference beam

(1.8) Cover for window (1.7)

(2.0) Pair of reflectors ("corner mirrors"): Reflect the light into itself over an angular range of about 12^0, enabling extremely simple adjustment of the setup.

(2.1) Large reflector in holder on stand rod (9 mm × 10 mm dia.); Mirror area: 160 mm × 180 mm (4 elements)

(2.2) Small reflector: Mirror area: 40 mm x 180 mm (1 element)

6.6.4 Setup and Adjustment

Coarse adjustment

- Set up the lens so that it is about 20 cm from the window(1.6). Make sure the midpoints of the window and the lens are at the same height.
- Set up the large reflector at the appropriate distance $s/2$ for the experiment: ~10 m.
- Set up the large reflector so that its center is at the level of the optical axis and its surface is perpendicular to this axis.

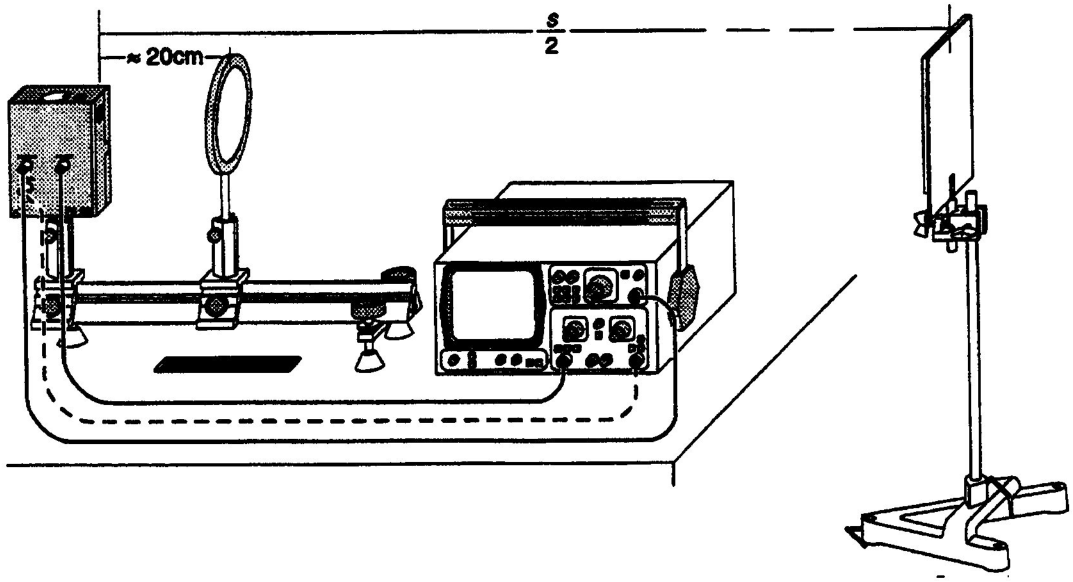


Figure 6.6: Setup for the speed of light measurement.

Fine adjustment, locating the signal

Switch on the operating unit (plug in the supply unit) and the oscilloscope as shown in Fig. 6.6. The PULSES output of the units should go to the Chan. 1 input of the oscilloscope; use a short cable for this to minimize broadening of the pulse. The Trigger Output should go to the External Trigger input of the scope, and the 10 MHz output can go to the Chan. 2 input. Set the oscilloscope to trigger on the leading edge of the Trigger pulse using the Trigger menu (Edge/External/Rising/Normal/AC) and trigger level knob.

To check the beam aiming, observe the corner reflector from a vantage point just above and just below the housing and lens. If the mirror is not illuminated or is only red at one edge, vary the position of the optical bench by turning it slightly and adjusting the height of the box until the center of the mirror is illuminated.

Set the small reflector over the top window to reflect some light into the photodetector. Start with the time scale at 10 ns/cm and a Channel 1 sensitivity ~50 mV per division. and the horizontal position so the pulse is toward the left side of the screen. The pulse from the reflected light should appear ~40 ns later. To see it more clearly, remove the small reflector and increase the vertical sensitivity if necessary. Once you see it, you can maximize its amplitude by adjusting the position of the lens and adjusting the aiming of the optical bench and/or the beam.

6.6.5 Absolute Measurement

Place the small reflector over the top window and adjust its position so that the two pulses have equal amplitude. Center the vertical cursors on the two pulses and read off the time difference.

Note: Keep your eyes open for small systematic effects that may affect the accuracy of your measurement. For best accuracy, it is probably best to measure to the centers of the pulses rather than the leading edges. Also you may find that the presence of the first pulse distorts the shape of the second; if so, you can remove the small reflector when setting the cursor on the 2nd pulse.

Repeat this for several reflector positions between about 3 and 15 meters. Realign after each move to maximize the amplitude of the reflected pulse.

- What is the best place on the box to measure distances from?

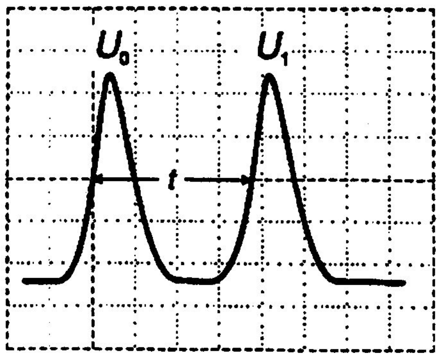


Figure 6.7: Absolute measurement of the transit time t of light for a path s from the time interval between measuring signal U_1 and reference signal U_0.

6.6.6 Relative Measurement

Start with the large reflector several meters away. Display the reflected pulse on Channel 1 and position it toward the left-hand side of the screen with the time scale set at 10 ns/cm. (See Fig. 6.7.) Position the time-measuring cursor over the center of the pulse and read off the time.

Now move the reflector by $\Delta s/2$, so that the light path is increased by Δs. Realign and determine the corresponding increase in the transit time Δt from the shift of the time signal (measured at its maximum as before; see Fig. 6.8).

Record multiple Δs_i, Δt_i-value pairs out to about 15 meters. Plot Δs *vs.* Δt and fit the data points with a straight line. The slope of the line should be the speed of light.

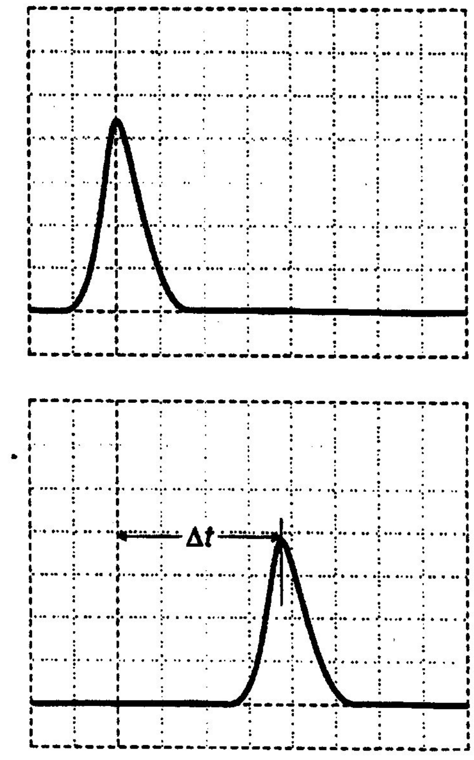


Figure 6.8: Relative measurement of the change in the transit time Δt of the light when changing the path by Δs. Top graph: Position of reflected pulse in first reflector position. Bottom graph: Position of pulse after changing the path by Δs

6.6.7 Measurement using Reference Signal

Set the reflector ~3 meters from the lens. Display only the reflected beam pulse on Channel 1 and move it toward the left side of the screen. Display the 10 MHz output on Channel 2. Use the phase adjuster knob(1.4) on the box to shift the 10 MHz signal so that the center of the reflected pulse is centered on the leading edge(or trailing edge) of the reference signal.

Then move the large reflector approx. 15 meter farther away. Realign to maximize the (small) signal pulse). Adjust the reflector position so that the reflected pulse is exactly 100 ns later and is now centered over the next reference signal pulse. Calculate the speed of light from the change in the position of the reflector.

Some additional questions …….

- Which technique gives the most accurate results?
- Do your results show unexpected deviations from c? If so, try to explain the origin of the systematic effect(s).

Experiment 6 - Speed of Light Apparatus list

1) CVI Michelson interferometer with He-Ne laser and beam expander
2) Tektronix Model 2022 200 MHz oscilloscope
3) Philips NIM pocket pulser
4) Leybold light speed measuring instrument
5) 50 mm diameter, 200 mm focal length achromatic lens
6) Large and small reflector
7) 50 mm lens holder assembly
8) Small optical bench
9) Assorted RG6-58C/U coaxial cables
10) 2 BNC tees
11) 2 BNC 50 Ω terminators
12) 4 BNC unions
13) BNC shorting stub
14) Hewlett-Packard 974A multimeter
15) RG6-58C/U cable scrap
16) Tape measure