Chapter 3 Geometric Optics

3.1 Introduction

Geometric optics is the technology that underlies most of the optical devices important to various aspects of our lives. These range from simple things like eyeglasses to complex instruments, such as the Hubble Space Telescope. A useful reference for this and subsequent experiments is Optics by F. Sears. A copy is available in the lab.

The basic physics is contained in two laws: For reflecting surfaces (Fig. 3.1), the angle of reflectance equals the angle of incidence,

$$\theta_i = \theta_r \tag{3.1}$$

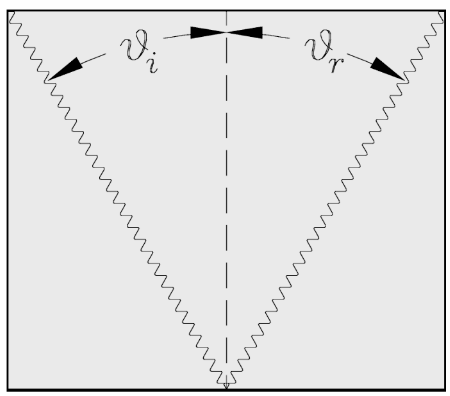


Figure 3.1 Reflected light rays

For refracting surfaces (Fig. 3.2), the angles of incidence and refraction are related by Snell's law:

$$n_i \sin\theta_i = n_r \sin\theta_r \tag{3.2}$$

where n_i and n_r are the indices of refraction in the respective media. Note that all angles are measured relative to the normal to the surfaces. In the geometric optics picture, a ray of light travels through a uniform medium in a straight line until a new interface is reached. The ray is deflected according to Equations 3.1 and 3.2, and this process is repeated until the light reaches its final destination. Such behavior is easily modeled on a computer, which has encouraged the development of complex lens systems that were unthinkable some years ago.

As you will learn in the Interference and Diffraction experiment, the rectilinear propagation of light is only an approximation; wave phenomena such as diffraction place important limiting bounds on the ultimate optical performances that can be obtained. This too can be computer modeled, but for most optical systems the geometric picture is already quite adequate. The purpose of this experiment is to familiarize you with the elementary principles described above and the basic constituents of optical design: lenses, mirrors and prisms. The experiment is divided into a number of separate chunks starting with a "ray projector" and two dimensional lenses and prisms. This will acquaint you with Snell's law of refraction and the qualitative behavior of optical elements. We will then explore further with measurements of real three-dimensional lenses on optical benches, followed by construction of simple telescopes. Finally, you will be asked to use an optical ray trace program to simulate some of the experiments you have performed.

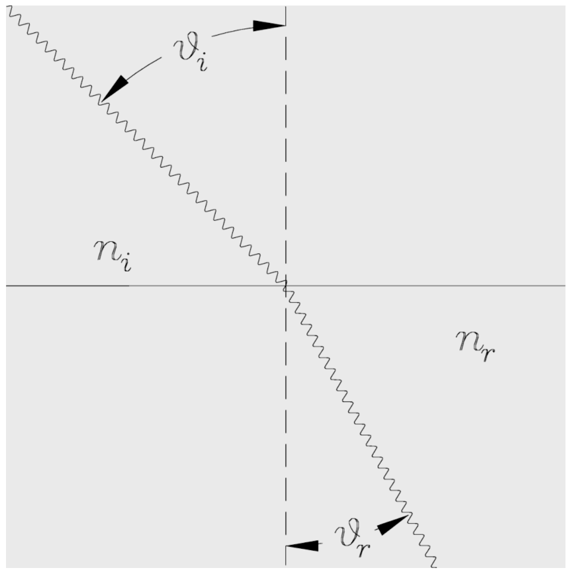


Figure 3.2 Refracted light rays

3.2 Two-dimensional Optics

The apparatus for these experiments consists of a "multi-ray" light projector and a number of acrylic lenses and prisms. The projector requires a low voltage power supply so you should use the same one that was part of the last experiment. The "diode box" is still required, but the voltmeters and ammeters are unnecessary, so that the wiring is quite a bit simpler. Operate the internal lamp at about 10 volts; higher voltages tend to heat and melt the plastic projector components. It also makes for painful encounters when you accidentally touch the metal parts with your hands.

The other components consist of a plastic tray that can be filled with water and various "two dimensional" optical elements. For these experiments, you will want to record the direction of rays by marking their position on paper taped to the bottom of the tray. Protractor and rulers will also be needed for these measurements.

For many of the investigations with the ray projector, you will find that the transverse width of all five rays is greater than the width of the plastic optical elements you are trying to test. This can make for lots of confusion so it's a good idea to turn the knobs on the projector box to remove those extra rays that aren't useful.

3.2.1 Snell's Law

The velocity of light at a given wavelength λ, *i.e.,* color, relative to the speed of light in vacuum (c) is given by the *index of refraction*, $n(\lambda)$, where

$$n_p(\lambda) = c/v(\lambda) \tag{3.3}$$

The subscript p stands for phase, to remind us that this is the speed that the phase of the light wave advances. There is another velocity called v_g or group velocity, that describes the speed of energy flow. These can be different (see Lab 6). Here we care about v_p. In most media, $n(\lambda)$ does not change rapidly with λ so we often approximate $n(\lambda)$ by a value, n, which is an average value for visible light ($\lambda \cong 580$ nm = yellow). If n is constant as λ is varied, then $v_p = v_g$. Typically $n = 1.5$ to 2.0 for dense optical media such as plastic or glass. When a light ray crosses the boundary between two media it will be deflected depending on the angle of incidence at the boundary and the indices of refraction of the two media on either side of the boundary. The specific relation, Eq. 3.2, is called *Snell's Law* (named after Willebrod Snell, who derived it in 1621).

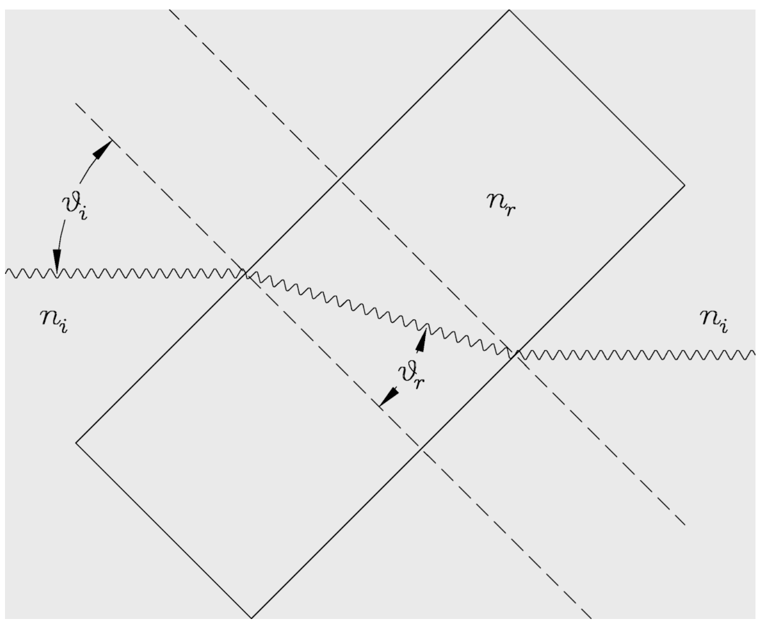


Figure 3.3 Geometry for measuring for Snell's Law

Both Snell's law and the law of reflection can be related to a single principle: Light rays take the path with the shortest time duration between two points.

- Set up the multiple-ray projector to produce a single ray. Adjust the low voltage power supply to obtain a sufficiently bright ray (about 10 volts should be sufficient).

<u>Careful: The ray projector will become hot!</u>

- Place the clear rectangular acrylic block (n = 1.49) at various angles relative to the incident ray and measure the angles entering and exiting the block. This is easily done by tracing the outline of the block and the rays on a blank sheet of paper. Angles in the range of 20^o to 60^o to the normal are good but you can try other angles as well.

- You can now connect the entrance and exit points and deduce the angles θ_i and θ_r. Compare your results with the predictions of Snell's law. (Recall that $n_{air} \cong 1.00$.) Does this relation also hold at the exit boundary? (The angles θ_i and θ_r and the indices n_i and n_r will be different from those for the entrance boundary, of course).

3.2.2 Total Internal Reflection

An interesting property of Equation 3.2 is that for light traveling from a medium having a large index of refraction to one with a smaller index, the light may be completely reflected rather than transmitted for large angles of incidence to the boundary (the angle where this just occurs is called the *critical angle*). This can be verified by sending a light ray through the equilateral prism and noting that at certain angles the ray is internally reflected, and does not exit the side of the prism. You can also observe that, just before the critical angle is reached, the transmitted beam is emitted almost parallel to the exit surface.

- Measure the critical angle, and from this calculate the index of refraction of the acrylic. Internal reflection can be viewed directly by picking up the rectangular acrylic block and looking through the small end. Can you see out the sides of the block? How do the sides of the block appear?

Total internal reflection, as observed above, is the basis for modern fiber-optic technology. Small-diameter glass fibers ($n \cong 1.5$) using total internal reflection can transmit voice, TV, and other signals over long distances with little attenuation using visible or infrared light. A graphic model of this can be seen at the end of the lab period.

3.2.3 Dispersion

We have assumed in the previous experiments that n_i and n_r are independent of λ. That this is not exactly the case could be seen using the equilateral prism from the previous exercise (see Figure 3.4). As you rotate the prism relative to the incident ray, you will notice that there is an angle of *minimum deviation* where the output ray direction doesn't change much as you rotate the prism further. The prism exaggerates the small differences in the bending angles due to the slight variation of the index of refraction of the acrylic plastic with λ. This is called *dispersion*.

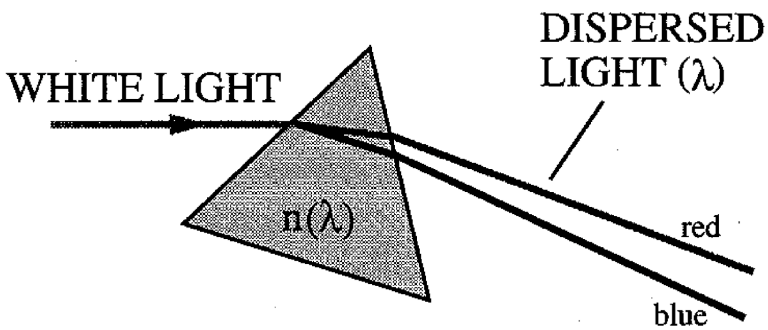


Figure 3.4 60° prism setup.

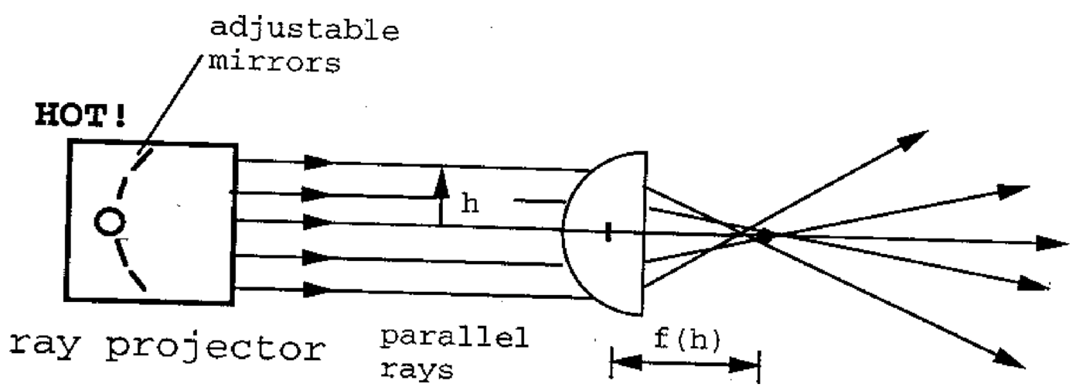
- Note and record the sequence of the colors of the dispersed light. Using Equation 3.2, determine which colors have the larger (and smaller) indices of refraction (the colors can be projected onto a white card or piece of paper for easy viewing). In Section 3.3, you will perform a more quantitative study of a prism and its dispersed spectrum.

3.2.4 Converging and Diverging Lenses

A suitable set of curved surfaces forming a boundary between different optical media can be arranged as a lens to collect and focus light. Normally the lens material has a higher index of refraction than the surrounding media, which is usually air, but as we shall see, this need not always be the case.

We can determine the *focal length* of a lens by determining the point where parallel rays, incident on one side of the lens, will come to a focus (Figure 3.4). Ideally all rays should focus to a common focal point independent of their distance from the axis, h, of the lens and independent of the wavelength of the light, λ.

Figure 3.5 Determining the focal length of a lens (positive focal length).



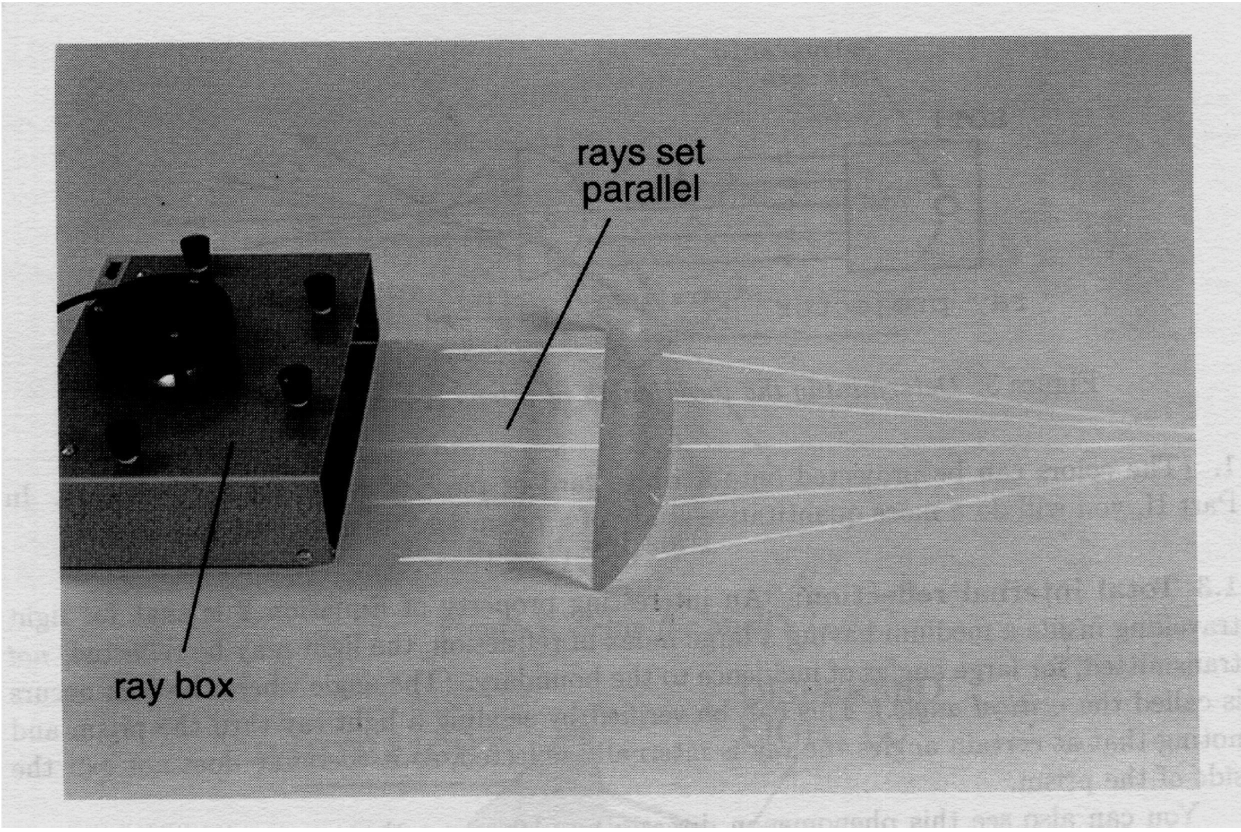


Figure 3.6 Single lens set up.

If a lens defocuses parallel light, but nonetheless forms a common "virtual" focal point (located on the same side as the entering light rays), the lens is said to have a *negative* focal length.

- Determine the focal length of the semicircular plano-convex acrylic lens (n = 1.49) as follows: Form a suitable set of parallel rays using the multi-ray projector by rotating the small adjustable mirrors inside the projector (see Figure 3.6). Determine that the rays are parallel to a few mm relative to the central, fixed ray. *Careful: The projector may be hot!*). Now place the lens *perpendicular* to the parallel rays and determine the focal point relative to the center of the lens. Orient the circular surface towards the incoming rays as in Fig. 3.7; otherwise internal reflection will block the two extreme rays from exiting.

- Note that the rays don't quite focus to a common point. Record the focal length as a function of the distance of the incoming rays from the optic axis, h. Also, even for a set of rays that appear to come to a common focus do the *red* and *blue* colors (wavelengths) come to exactly the same focus?

- Does the focal length change significantly if you turn the lens around? Knowing Snell's Law, how would the focal length change if one or both radii were increased or decreased, or if the index of refraction of the surrounding medium increased or decreased? (Recall that your eye has a liquid behind its lens).

The slight differences (*i.e.*, aberrations) in the focal lengths you noted above are the result of the non-ideal shape of the lens (*e.g.,* circular versus parabolic) and the fact that *n,* as we've noted, varies slightly as a function of λ. These two effects result in *spherical aberration* (SA) and *chromatic aberration* (CA) respectively.

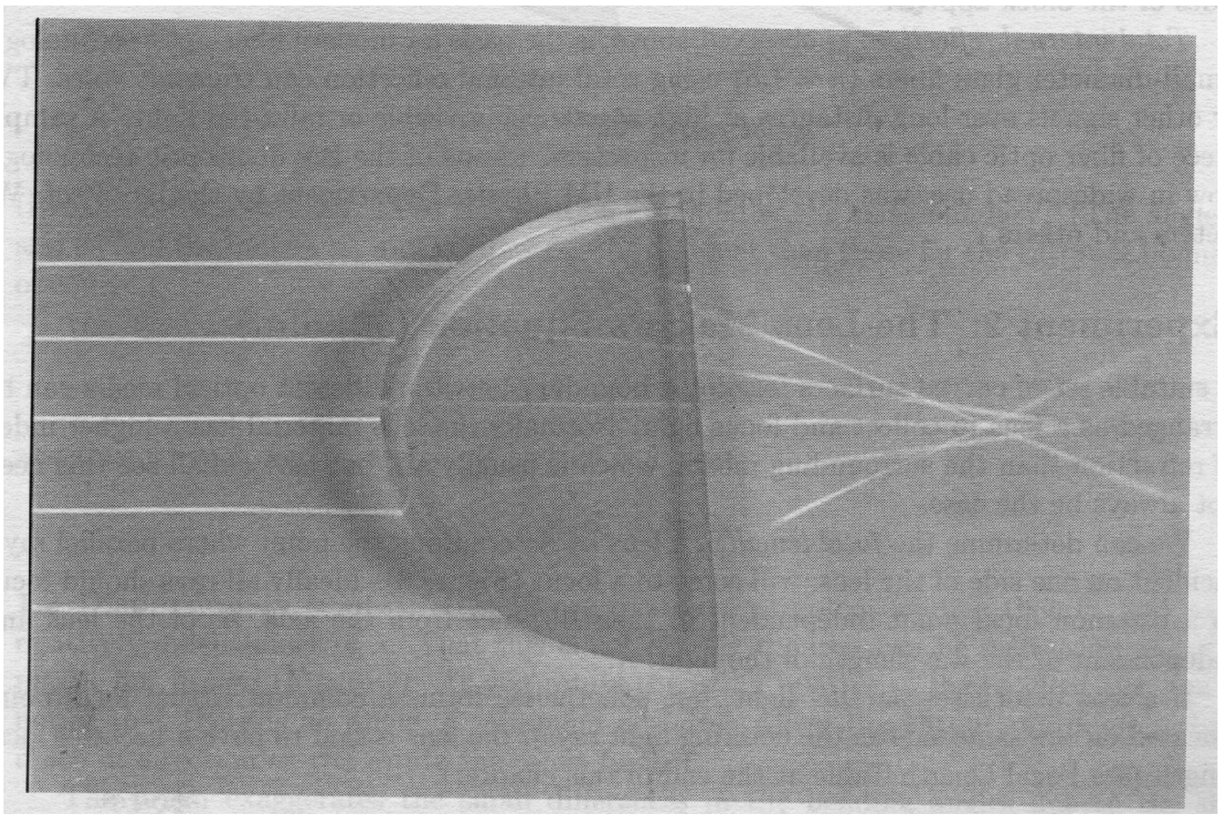


Figure 3.7: Semi-circular lens used to observe spherical aberrations.

Good cameras, microscopes, telescopes, *etc.* are expensive because they have multi-element precision lenses that are designed to have little or no aberrations and are coated with anti-reflection films. (In contrast, the "disposable" cameras sold in grocery stores will likely not have a well-corrected optical system.)

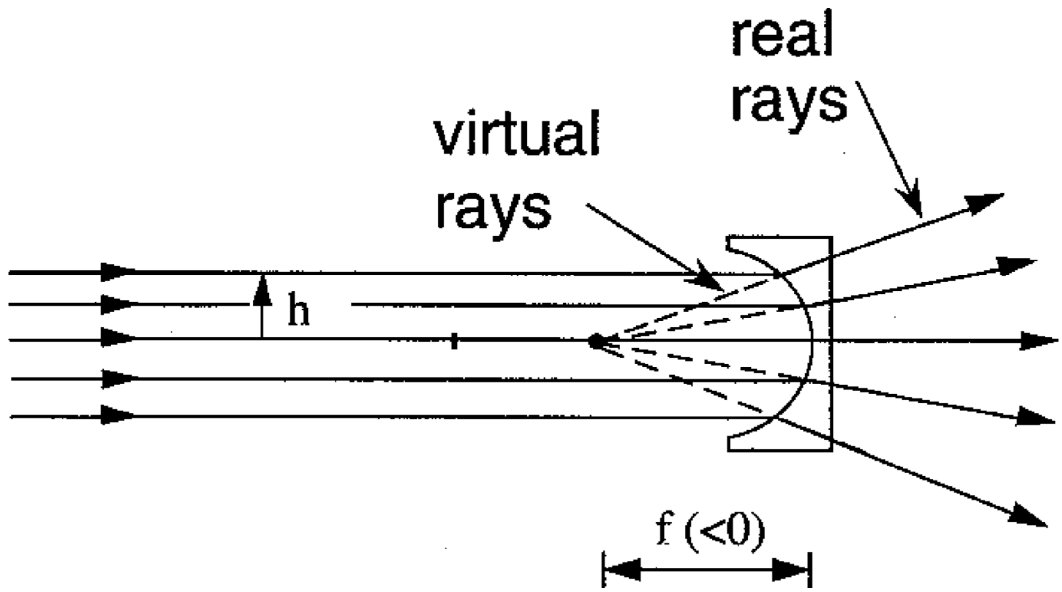


Figure 3.8 Determining the focal length of a defocusing lens (negative focal length)

- You can observe (and measure) the focal length of a *diverging* lens (plano-concave or bi-concave) in the same manner. Now, of course, you must trace the diverging rays back to a negative focal point (see Figure 3.8) (Note: You may also see faint, *real* rays in the region of the *virtual* rays. These are reflections and are *not* the virtual rays). How does the focal length compare with that of the plano-convex lens of the same curvature? Note and record the appropriate information in your notebook.

- Now observe the focal length of the above two lenses in *combination* as a function of their separation, d (see Figures 3.9 and 3.10). Start with $d \cong 0$. (*i.e.* both lenses fit together). Where is the focus?

- Now separate the lenses and qualitatively observe how the focal point depends on the separation of the two lenses. Also note whether the combination has a net positive or negative focal length, and how this depends on the separation of the lenses. What happens if d equals one of the focal lengths, *i.e.,* one lens is placed at the focal point of the other lens (Figure 3.9).

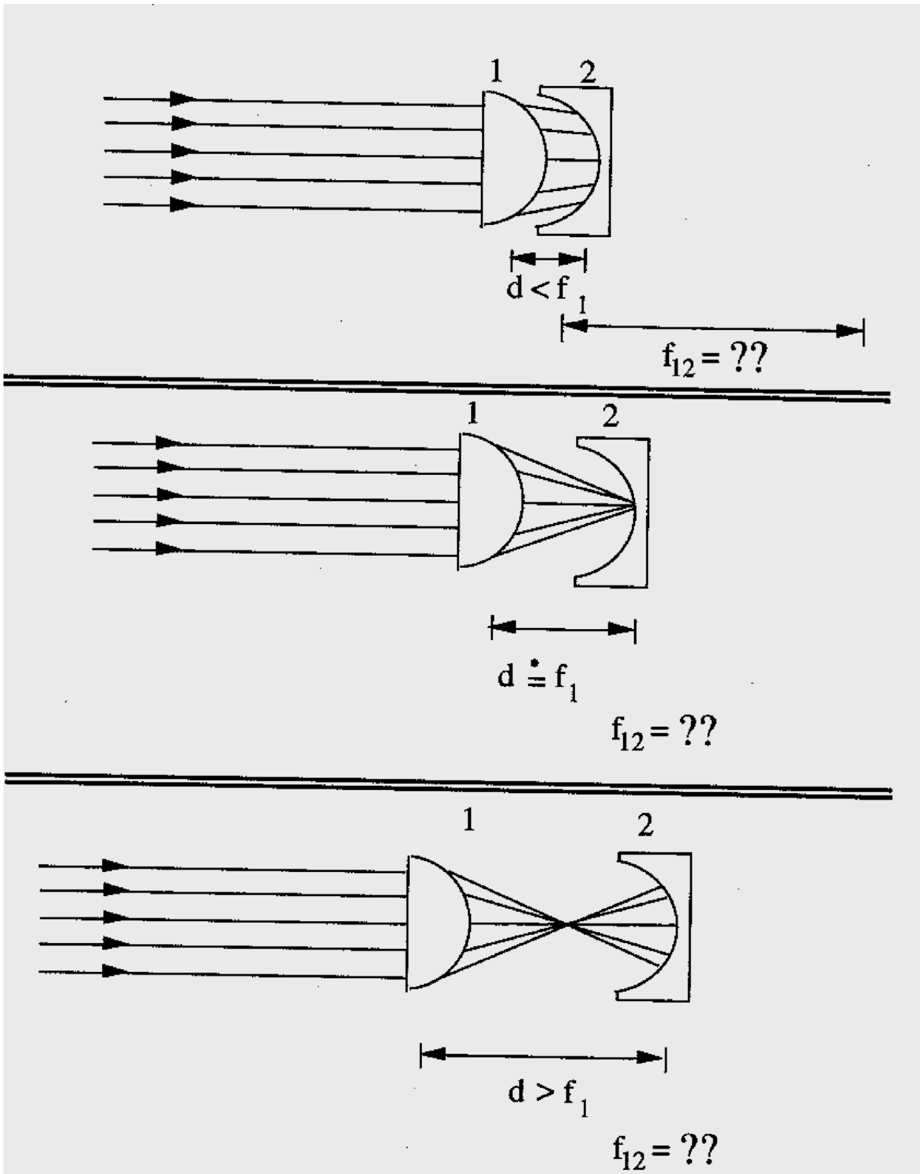


Figure 3.9: Possible combinations of thin lenses

One method to correct for chromatic and other aberrations in lenses is to combine several lenses having different focal lengths and indices of refraction into a compound lens. Thus a well-corrected compound lens may have ten or more elements and is expensive to make.

- Is a convex lens *always* focusing? Is a concave lens *always* defocusing? What if the surrounding media had a *higher* index of refraction than the lens does? As an example, suppose the lens is made from water (n = 1.33) surrounded with acrylic plastic (n = 1.49). What do you expect for the focal length? An example of a lens like this is a column of heated air surrounded by cool air. This is the basis for the *mirages* you see near a hot roadway.

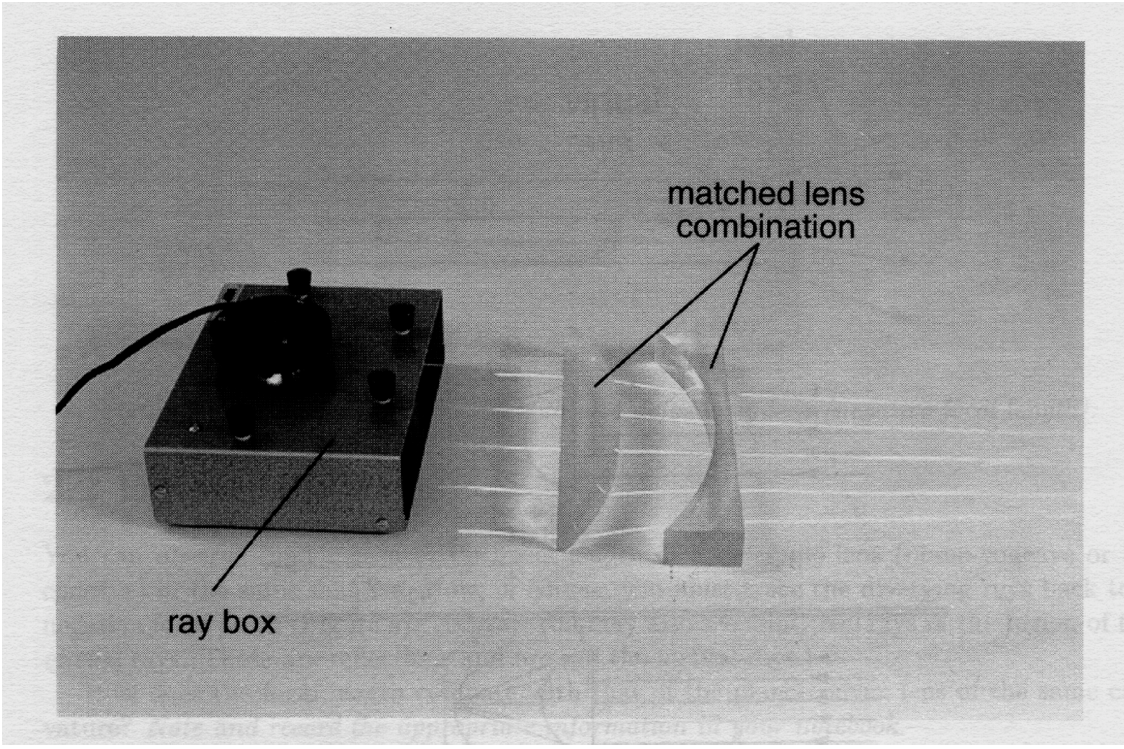


Figure 3.10: Combination of lenses

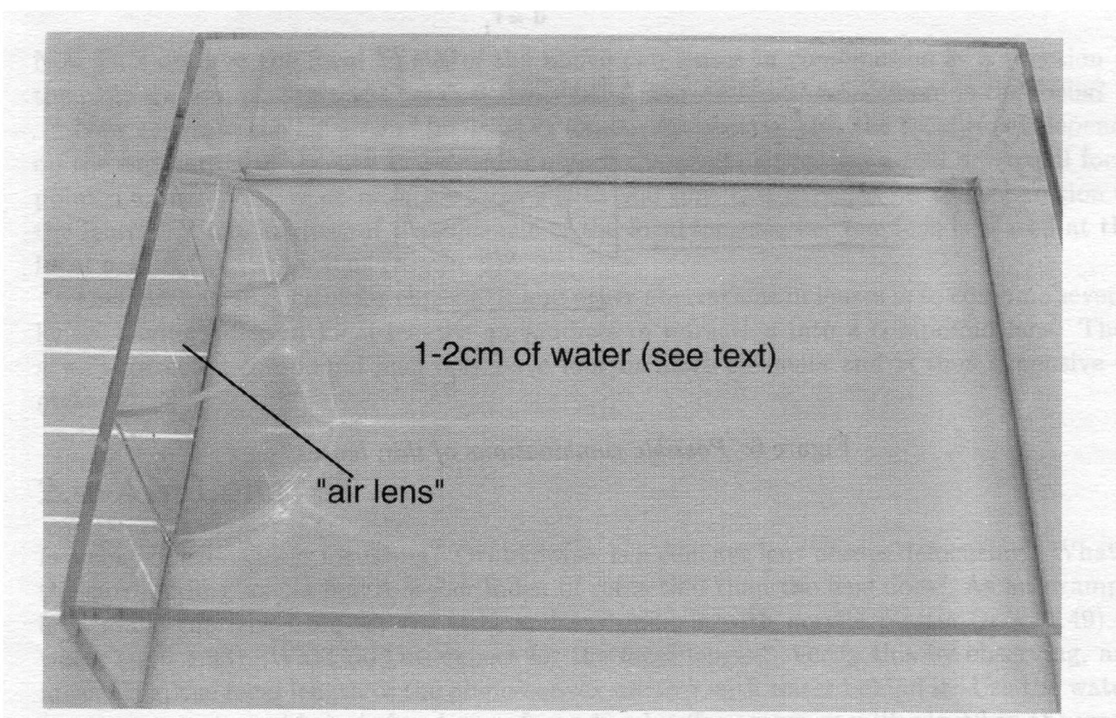


Figure 3.11: Water tray and "air lens"

3.2.5. Fiber Optics

You have already seen that total internal reflection can occur at the boundary of a dense optical medium such as glass or plastic. As long as the surfaces are clean and the material is transparent, the number of reflections can be very large. Light can be injected at one end of a cylindrical fiber and transported for many kilometers before attenuation requires an active device to detect and amplify the signal. Try this with a few feet of unclad fiber and a small He-Ne laser. Another example is demonstrated with a plastic water bottle with a hole near the bottom. Align the laser beam to enter the glass window and exit via the open hole. Then fill with water (use the stainless steel beakers to catch the outflow). Mix in a small amount of milk to visualize the light path better (incidentally, what is the mathematical shape of the water stream? Why?).

3.3 Three-dimensional Optics

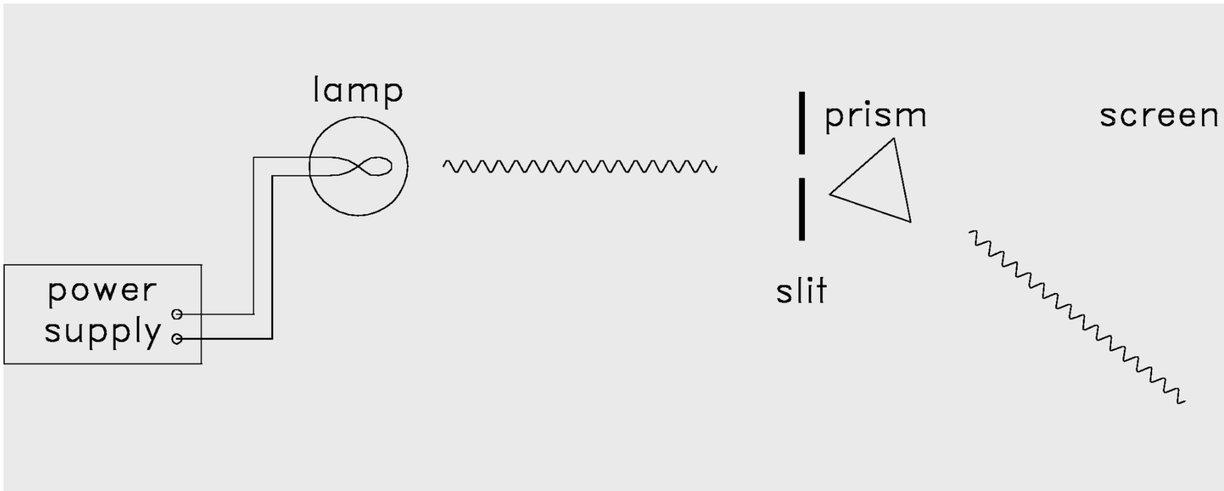
In the following experiments, you will use the optical bench and associated accessories to explore optics in the paraxial limit. This corresponds to situations where the angles of rays with respect to the axis of symmetry are small enough that the approximation $\theta \cong \sin\theta \cong \tan\theta$ is valid. It is also possible to sketch the behavior of these rays with simple geometric constructions using only a ruler. These drawings are sufficient to find the focal properties of a system and its magnification.

Although the paraxial approximation seems like a fairly severe restriction, it is a very good way of describing the behavior of an optical system in the simplest possible terms. If your goal is to find the important characteristics of an optical instrument, this is the place to start. The notion of using a ruler and paper to "compute" a design may seem a bit crude. For the more mathematically inclined, there is a matrix formalism that models the geometric method exactly. As might be expected, the matrix method is particularly useful for numerically exploring variations in parameters such as focal length, separation of lenses, *etc.* A more complete description of the matrix method is beyond the domain of this course, but it can be found in any book on introductory optics.

3.3.1 Measuring the Index of Refraction

The index of refraction of a material can be precisely measured by the deflection of a light beam. Arrange the low voltage lamp, the light baffle with slit, a prism and a white screen as shown in Figure 3.12 below. The refracted beam is deflected through a fairly large angle so you will need to extend the white screen with a length of white poster board held by a small C-clamp.

Figure 3.12: Arrangement to measure the refractive index of a prism



As you slowly rotate the equilateral prism around a vertical axis, the spectrum will first move inward, then outward again. There is a minimum angle of deflection given by

$$\delta = 2\sin^{-1}\left(n\sin\frac{\alpha}{2}\right) - \alpha \qquad \text{or} \qquad n = \frac{\sin\left(\frac{\alpha+\delta}{2}\right)}{\sin\left(\frac{\alpha}{2}\right)} \tag{3.4}$$

where n is the index of refraction of the prism (assuming $n_{air} = 1$) and α is the vertex angle of the prism which in this case is 60°. Measure the minimum deflection and compute the index of refraction. Compare the indices for red and blue light.

3.3.2 Focal Properties of Lenses

The paraxial approximation of geometric optics was stated earlier as a limit on the angles of permissible rays. This is usually accompanied by another assumption, the thin lens approximation. which is valid if all lenses are thin compared to the lens separations. Under these circumstances, a lens with focal length f will transform rays from an object at position s_o to an image at position s_i with a relationship given by the Gaussian lens equation:

$$\frac{1}{f} = \frac{1}{s_o} + \frac{1}{s_i} \tag{3.5}$$

In the simplest case, if the object is very far away from the lens, $1/s_0 \approx 0$ so the image distance is the same as the focal length. This makes finding the focal length of converging lenses relatively straightforward.

- Measure the focal length of two convex lenses, using the low-voltage lamp as a light source. Set $s_0 \approx 1$ meter and find s_i by adjusting the position of a white screen to obtain the sharpest image.

- Explore the Gaussian lens equation by varying s_o over the range from 25 *cm* to 150 *cm* and measuring the corresponding s_i. Compare the predictions of the equation with your experimental values. Are there limits to the values of s_o which permit a real image to be formed and detected?

- Investigate the chromatic aberration of the shorter focal length lens by finding the focal length for red and blue rays selected with interference filters. For this exercise, set $s_0 \approx 2f$ and compute the exact focal length for each color from the corresponding value of s_i.

- Devise a way to estimate the focal length of the concave lens and do so.

3.3.3 The Lensmaker's Formula

As you might expect, the focal length of a lens depends on both the curvature of the surfaces and the index of refraction of the optical material. In the paraxial thin lens approximation, this is expressed by the "lensmaker's formula":

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \tag{3.6}$$

where n is the index of refraction and R_1 and R_2 are the radii of curvature. The values, R_1 and R_2, have signs attached to them ; the convention is shown below:

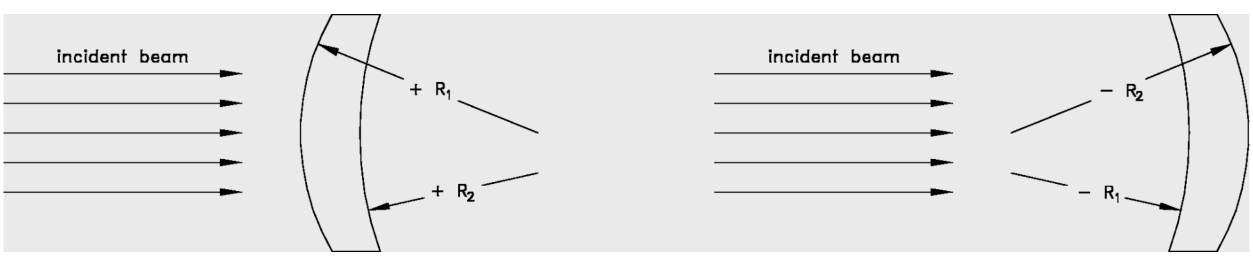


Figure 3.13: Sign convention for lens curvature

If R_1 and R_2 are equal, then, according to Equation 3.6, f is infinite, *i.e.,* equivalent to looking through a piece of window pane.

Determining the radius of curvature of an optical surface is a matter of geometry. As shown in Figure 3.14, the sagitta of an arc of a circle is related to the radius by the Pythagorean theorem:

$$(R-s)^2+\left(\frac{d}{2}\right)^2=R^2 \tag{3.7}$$

Solving for R in terms of s:

$$R=\frac{1}{2}\left(\frac{d^2}{4s}+s\right) \tag{3.8}$$

A device for measuring the sagitta of an arc is called a *spherometer*. The ones available in the lab have $d = 40\ mm$, and can be used to determine s. Measure the radii of curvature of both surfaces of all 3 lenses and the mirror. Compute their focal length from Equation 3.6. Use 1.50 for the index of refraction.

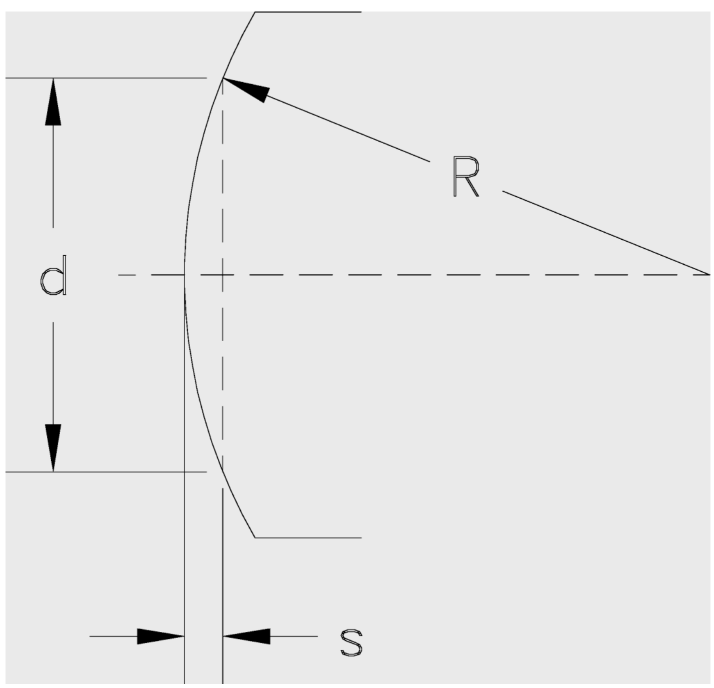


Figure 3.14: Sagitta and the radius of curvature of a lens

3.3.4 Spherical Mirrors

Under many circumstances, mirrors are more suitable optical elements than lenses. This is particularly true when the diameter must be large as in an astronomical telescope, for example. An additional advantage is the achromatic behavior of reflection since the focal length of a curved mirror is independent of wavelength (the downside is that monochromatic aberrations such as coma and astigmatism are not so easy to compensate).

For mirrors, the formula corresponding to the lensmaker's formula is:

$$\frac{1}{f} = \frac{2}{R} \tag{3.9}$$

- Test this relationship by measuring the focal length of the spherical mirror and compare with the estimate obtained from Equation 3.8 and 3.9. To find the focal length, tape some white paper to the back of the aluminum baffle just to the side of the 1" hole. Move the screen or mirror support until the sharpest image is found.

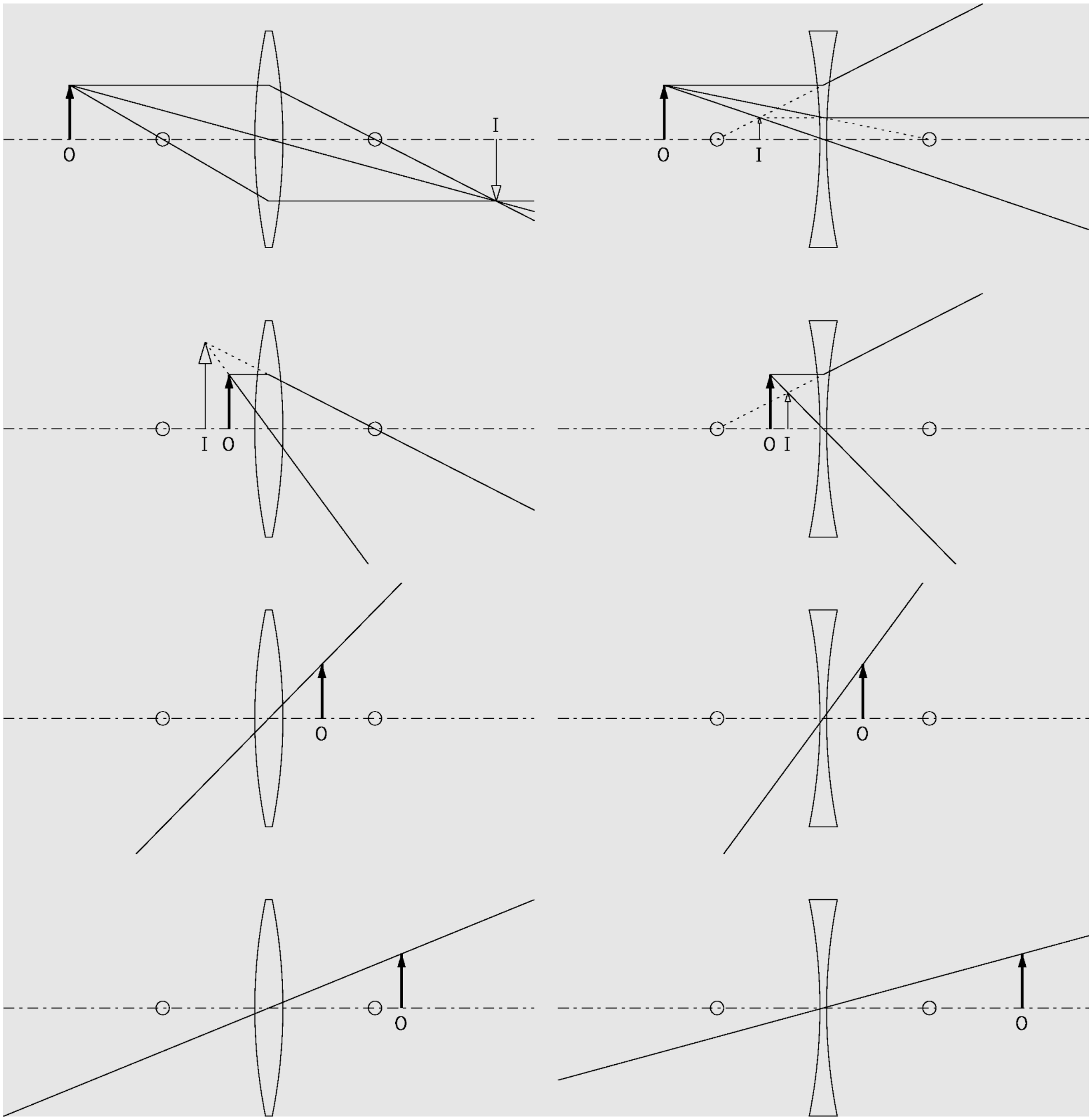


Figure 3.15: Graphical ray tracing diagrams

3.3.5 Graphical Ray Tracing

The behavior of a system of thin lenses can be graphically understood by a fairly simple geometric construction. The basic idea is to trace two or three rays from a representative element of an object through the optical system to see what kind of image is produced. The image location and magnification are usually the parameters of most interest.

By convention, optical systems are sketched with the incident rays traveling from left to right. The rules for graphical ray tracing are as follows:

1. A ray through the center of a lens is undeflected (in the limit of a thin lens, the ray must enter and exit at the same angle at the optic axis).

2. For a converging lens (positive focal length), a ray through the near side focal point will exit the lens parallel to the optic axis. A ray parallel to the optic axis on the near side will exit the lens and pass through the far side focal point.

3. For a diverging lens (negative focal length), a ray aimed at the far side focal point will exit the lens parallel to the optic axis. A ray parallel to the optic axis on the near side will exit the lens as if it had emanated from the near side focal point.

4. Systems of lenses can be treated by iterating the procedure described above with the image of the proceeding lens becoming the object for the next. This works even if some of the images are virtual. Be careful to observe a consistent sign convention. (See Fig. 3.13.)

Figure 3.15 shows all possible configurations of object positions relative to converging and diverging lenses. Some of the rays are missing. As part of your assignment, construct these missing rays and include with your lab report.

3.4 Telescopes

Optical telescopes are interesting examples of more complicated systems with several optical elements. For an instrument that will be used to aid human vision, the basic requirement is to accept an essentially parallel bundle of rays from a distant object and transform them to an output bundle of parallel rays which can be viewed by the eye. From graphical ray tracing it is easy to show that the angular magnification is given by

$$m_{ang} = -\frac{f_{obj}}{f_{eye}} \tag{3.10}$$

where f_{obj} and f_{eye} are the focal lengths of the objective and eyepiece lenses respectively. For the magnification to be large, you need to make the focal length ratio as big as possible. The easiest way might be to decrease the eyepiece focal length but this creates mismatch problems with the eye for distances less than one inch (high power telescopes for mice would be much easier to fabricate!).

In this section of the experiment, you will assemble two kinds of refracting telescopes. The difference will be the type of eyepiece lens. With a negative focal length, the telescope design is named after Galileo because it was he who first built such an instrument and got into trouble for finding out more about our solar system then was deemed proper at the time.

With a positive focal length eyepiece, the system is usually called an astronomical telescope for lack of a better name. From Equation 3.10, we see that the angular magnification will be negative so the image is inverted which is annoying for viewing terrestrial objects. High power binoculars with magnification of 6 or more get around this problem by inserting complex prisms to restore the image orientation by canceling the inversion.

For the objective lens, use the 200 *mm* focal length double convex lens and the positive and negative 50 *mm* lenses for the eyepiece for the astronomical and Galilean telescopes respectively. To make this setup somewhat easier to test, a shorter, lighter optical bench is provided to hold the lenses at proper separation.

- Find the separation of objective and eyepieces that produce a good focus for a distant object. By looking at marks on the white boards at the end of the room, estimate the magnification and compare with Equation 3.10. Include in your lab report a ray diagram showing where the focal points of the two lenses are and the image position. Compare the optimum lens separation you found with what you would expect from the ray diagrams.
- Compare the fields of view of the two telescopes.

3.5 Computer Simulation of Geometric Optics (if time permits)

The availability of fast computers has radically changed how optics design is performed. Analytical techniques have long been available to reduce or remove aberrations by explicit expansion of the focal errors in terms of incident angle and displacement from the optic axis. These classical techniques become unmanageably complex for higher-order errors. The computational method used by the computer programs is brute force – you tell an optimization program which parameters it can vary, such as surface curvature and spacing, and the code then fiddles until an acceptable solution is found or it gives up trying.

The program available for this class is called "Optics Lab". It runs on the PCs in the lab and will simulate a very wide variety of optical designs. However, it does not include a search algorithm for optimization so this program is inappropriate for developing a new design. There are three basic elements which you will need to manipulate: a source of light, various kinds of optical components such as lenses, mirrors, prisms, slits, *etc.,* and screens where the profile of the light distribution can be visualized. Click on the element to set it up. The source can be manipulated to produce a converging, diverging or parallel bundle of rays and the colors of light can include six different wavelengths. The refractive components can be completely specified in terms of their geometric shapes and constituent glasses. For your simulations, select BK7 as the standard material for lenses and prisms. The screen facility can display the beam profile under many different circumstances. It is less confusing to set the software switches so that rays are calculated only at the manually set screen locations.

- Arrange a parallel beam of light with red, green and blue rays to pass through a slit. Place a prism downstream and perform the same exercise in software as was done in Section 3.3.1. By using a similar scale, you should be able to reproduce your experimental geometry and measurement.

- Simulate the measurements performed in Section 3.3.2 with the 50 mm and 200 mm focal length lenses. Use the screen facility to determine the best focused image position and note how wide the spot size is under various conditions. Find the focal point separately for red and blue wavelengths and compare with your measurements at 650 and 450 nm.

- Single lenses cannot produce very good images for a number of reasons you have explored. The next most complex arrangement is a doublet of two lenses cemented together. By choosing glasses with different dispersions it is possible to eliminate a considerable fraction of chromatic aberration while simultaneously reducing the other troublesome monochromatic errors such as spherical aberration and coma. To make a 200 mm focal length doublet, use the following parameters in the appropriate menu:

Lens 1:	R_1	=	114.73
	R_2	=	-81.92
	t	=	9.8
	d	=	50.0
	glass	=	BK7
Lens 2:	R_1	=	-81.92
	R_2	=	-354.28
	t	=	3.3
	d	=	50.0
	glass	=	F4

  where all dimensions are in millimeters (*t* is the lens thickness at the center and *d* is the diameter). Compare the image spread obtainable with this design to that for the biconvex one used earlier.

- Simulate the performance of the telescopes you assembled in Section 3.4.

- A number of files are available which contain descriptions of all sorts of telescopes, camera lenses, *etc.* There is also one that simulates the human eye. Look at some of these to see how they perform.

- Experiment with various components and see what kind of systems you can design.

Experiment 3 - Geometric Optics
Apparatus list

1. Water tray
2. Two-dimensional plastic lenses and prism (6 pieces)
3. Multi-ray projector
4. Hewlett-Packard model E3632A low voltage DC power supply Metric tape measure
5. Optical bench assembly
6. Five optical bench mounting clamps
7. Two lens holders
8. Tungsten lamp and holder
9. Light baffle
10. Focusing screen
11. Mounted glass prism
12. Short optical bench assembly
13. 20 cm focal length converging lenses
14. 5 cm focal length converging lens
15. 5 cm focal length diverging lens
16. Set of three interference filters
17. Aluminum slit
18. Fiber optic filament
19. He-Ne laser
20. Water bottle with spigot
21. Skim milk
22. Ruler
23. Protractor