Radioactivity

Carl W. Akerlof
January 4, 2011

Overview

For most of you, this assignment will be your first experiment in Physics 441/442. It will introduce you to various aspects of radioactivity including detection, interaction with matter and some aspects of the underlying nuclear physics. Unlike many other kinds of measurements, the statistical nature of counting experiments is described accurately by the Poisson distribution. Thus, it is an ideal opportunity to test various statistical techniques that will be useful throughout experimental science. In outline, the work will include the following tasks:

1. Measure the counting rate of a Geiger tube as a function of high voltage to determine the best operating point.

2. Determine the Geiger tube deadtime by measuring counting rates as a function of flux intensity.

3. Measure background rate and show that it is consistent with a Poisson distribution.

4. Measure the range in air for alpha-particles from ^{210}Po decay.

5. Measure the half-life of ^{210}Po from the change in count rate over a two-week interval.

6. Measure the range of β^- particles in polyethylene and aluminum for decays of ^{90}Sr and ^{204}Tl. Show that the energy losses scale approximately with the mass of material traversed and compare to measurements for α particles.

7. Measure the radioactivity of potassium chloride due to the decay of ^{40}K and determine the disintegration rate of your body in microCuries.

8. Measure the attenuation coefficient for γ-rays from ^{60}Co and ^{137}Cs. Examine the statistical quality of your fit to test if attenuation is adequately described by an exponential dependence on absorber thickness.

Historical Background

The discovery of radioactivity by Henri Becquerel in 1896 is an example of a serendipitous event that rewards the prepared. Becquerel was interested in exploring the fluorescent behavior of certain uranium salts by exposing the crystals to sunlight and then measuring the re-emitted light with photographic plates. In these experiments, he noticed that darkening of the plates took place even when the uranium salts were not initially bathed in light. Within a decade, Marie & Pierre Curie, Ernest Rutherford, Paul Villard and others had shown that the spontaneous decay of uranium and other elements is accompanied by the emission of alpha, beta and gamma-rays, corresponding respectively to ^{4}He nuclei, electrons (or positrons) and photons, with energies ranging from 10's of keV to a few MeV. These processes are mediated by three of the four fundamental forces of physics: the strong interaction (α), the weak interaction (β) and electrodynamics (γ). Many of the characteristics of these radiations will be explored in this experiment.

Detection of nuclear radiation depends entirely on electromagnetic effects. For charged particles such as α and β, the passage of a fast charge through any material will accelerate atomic electrons to energies sufficient to cause ionization or excite atomic transitions that radiate light. Particularly for energies less than an MeV or so, γ-rays interact with atoms via the photoelectric effect and eject electrons with energies close to that of the original photon. Such photons can then produce ionization or luminescence by the same processes outlined for α and β-rays. For higher energy photons, Compton scattering and pair production become important which, although different in detail, produce similar ionizing effects in materials.

Detecting single particles from radioactive decay requires a bit of subterfuge. Consider the ionization signal generated by a 100 keV beta passing through a neutral gas. It will take about 20 eV for every ion pair created so the total available charge from this event is 5000 electrons or 8×10^{-16} Coulombs. Assuming this charge is collected by some electronic circuit, the voltage response will be given by $V = Q/C$ where C is the circuit capacitance. For discrete element circuits, typical input capacitances are of the order of 10 picofarads, constraining the signal voltage to less than 100 microvolts. This is too low to easily distinguish from random noise, especially with the measurement equipment of the early 20th century. The solution was invented by Hans Geiger in 1908 and subsequently improved with the help of Walter Müller in 1928. By allowing the ionization to occur in an inert gas in a high electric field, the liberated electrons can gain enough energy to initiate further ionization, greatly amplifying the charge collected on the electrodes. In this so-called Geiger regime, the particle avalanche develops until the accumulation of positive space charge from the slow-moving gas ions reduces the accelerating field near the anode. On longer time scales, the positive ions migrating to the cathode can recombine at the walls to liberate ultra-violet photons that may generate further cascades. Detector recovery requires that such processes must be eliminated. This is usually performed with the addition of a ~10% admixture of a halogen such as Cl_2 or Br_2. The halogen molecules donate electrons to the positive noble gas ions but do not contribute UV photons when they, in turn, are neutralized at the cathode surface. For a more extensive description, see Knoll, chapter 7.

Experimental Equipment

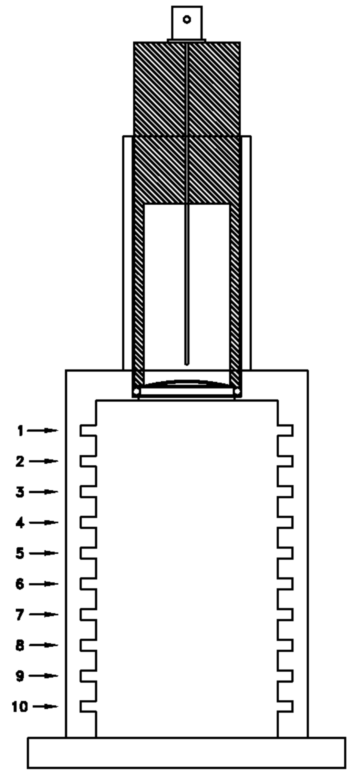

Figure 1. The Geiger counter and support assembly. The numbers 1 – 10 label slots for inserting sources and absorbers.

Figure 2. The Spectrum Techniques ST360 Radiation Counter. (from www.spectrumtechniques.com/st360.htm)

The Geiger tube and support stand for this experiment are shown in Figure 1 above. The tube has an inner diameter of 1.1″ and an effective length of 2.4″. The thin mica end window has a nominal thickness of 1.9 ± 0.1 mg/cm^2. Since air has a density of 1.2 mg/cm^3 at 20° C, this corresponds to an air path of about 1.5 cm, a fact that will be needed later when investigating the range of α particles. **Do not touch the thin end window – it is very fragile.** For more details, check the manufacturer's Web pages; the tube is a Canberra P2131 (see www.canberra.com/products/463.asp).

The Geiger tube must be connected via coaxial cable to the Spectrum Techniques ST360 Radiation Counter (or its equivalent). The counter electronics have provisions for supplying the Geiger tube high voltage, counting the radiation pulses and controlling the count integration time. These various functions can be accessed sequentially by toggling the *Display Select* button.

Experimental Procedure

Determine Geiger tube operating voltage: The minimum voltage for achieving sufficient gas multiplication is about 800 volts for the tubes used in this experiment. To determine the optimum operating point, start at about 700 volts and increase in increments of 40 volts to trace out a curve similar to Figure 3. Use a ^{90}Sr source to provide a reasonable flux of particles and count for long enough to determine the curve shape to 2% accuracy or better. Stop increasing the voltage when the rate appears to rise perceptibly above the previous points as shown below. **In any case, do not apply greater than 1200 volts.**

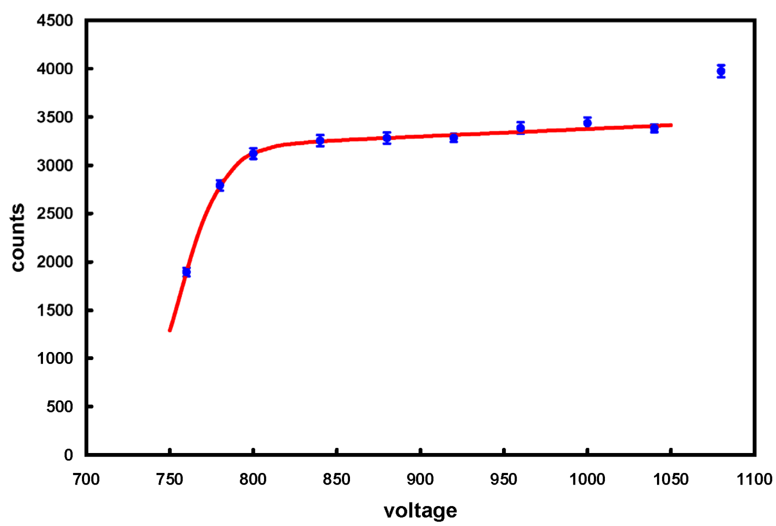

Figure 3. HV curve for a P2131 Geiger tube. The operating voltage was set at 920 v.

Determine the Geiger tube deadtime: The Geiger tube signal amplification process guarantees a minimum deadtime, τ, when the tube is insensitive to another pulse. If $\dot{N}$ is the intrinsic particle rate and $\dot{N}'$ is the corresponding measured rate, the relation between the two is:

$$\dot{N}' = \frac{\dot{N}}{1+\dot{N}\cdot\tau}$$

Note that in the limit, $\dot{N} \to \infty$, $\dot{N}' \to 1/\tau$. For the tubes used in this experiment, τ is of the order of 0.2 ms so 10% effects will not be apparent unless $\dot{N}$ is larger than 500 Hz. To measure the deadtime, you will count two half-circle ^{204}Tl sources, first one at a time and then both together. With fairly obvious notation, the value for τ can be obtained from the following equation:

$$\tau = \frac{\dot{N}_1' + \dot{N}_2' - \dot{N}_{12}'}{\dot{N}_1' \cdot \dot{N}_2' + \sqrt{\dot{N}_1' \cdot \dot{N}_2' \cdot (\dot{N}_{12}' - \dot{N}_1') \cdot (\dot{N}_{12}' - \dot{N}_2')}}$$

To check the validity of the above equation, perform these measurements for various counting rates by using three different source-to-counter distances (put source trays on levels 1, 2 and 3). Make at least three sets of measurements for every specific counter distance. To obtain reasonable accuracy, you will need individual counts of at least 50,000. Estimate the statistical error based on Poisson distributions for the measured counts. This is technically a somewhat messy computation for which you may want to use *Mathematica* or a similar symbolic algebra program. Compare the statistical prediction with the values you have obtained for τ.

Compare the background count rate with the Poisson distribution: The background rate for this equipment in the Physics 441/442 lab should be about 0.7 Hz. Set the counter timer for 3 seconds and record each of the counts for 100 independent measurements. Compute the mean count in the 3-second interval and its variance. The Poisson distribution predicts the two to be equal. Is that prediction verified within the predicted error for the variance? Create a histogram of the number of observations with 0, 1, 2, … counts and compute the corresponding expected numbers based on Poisson statistics. Compute the χ^2 for the comparison of the histogram and the Poisson expectation. Is this number consistent with the assumption of Poisson statistics?

Measure the range of ^{210}Po alpha particles in air: In general, nuclear decay occurs whenever the sum of the byproduct masses is less than the parent. The ^{4}He nucleus is exceptionally well-bound so that heavy nuclei such as Po, Th, U, Pu, etc. usually decay by alpha emission whereas other possibilities such as emitting single neutrons or protons are either rare or non-existent. Even then, the presence of a large potential barrier within the nucleus restricts the alpha particle energies to a narrow range around 5 MeV. If the energy is lower, the decay lifetime becomes essentially infinite; if higher, the decay occurs so quickly that the parent disintegrates immediately. In this part of the experiment, you will observe the very rapid energy loss of alpha particles as they travel through air.

Although precise calculations of charged particle energy loss are complicated by the atomic structure of matter, the essential behavior can be easily understood. First of all, assume that the incident ionizing particle has a kinetic energy greater than any of the atomic electrons that it encounters. Since the interaction is basically electrostatic, the force between an atomic electron and the ionizing particle is proportional to ze^2 where z is the number of units of charge of the incoming particle. For an alpha particle, $z = 2$. During a short time interval, Δt, an atomic electron will gain a momentum, $\Delta p = F{\cdot}\Delta t$. The duration of this interval is roughly given by the diameter of an atom divided by the incident particle velocity, v. The energy imparted to an electron is $\frac{1}{2}p^2/m_e$, proportional to z^2e^4/m_ev^2. It is convenient to replace e^2 by the classical electron radius, r_e, which is defined in MKS units as:

$$r_e = \frac{e^2}{4\pi\varepsilon_0 m_e c^2}$$

With this substitution, the energy loss of an ionizing particle is predicted by the Bethe-Bloch formula:

$$-\frac{dE}{dx} = \frac{4\pi m_e c^2 r_e^2 z^2 n_e}{\beta^2}\left[\log\left(\frac{2m_e c^2\beta^2}{I\cdot\left(1-\beta^2\right)}\right) - \beta^2\right]$$

where $\beta = v/c$, n_e is the electron density of the absorbing material and I is the mean ionization energy for the stopping medium. For an alpha particle with a kinetic energy of 5.3 MeV, the estimated ionization loss rate will be about 1400 times greater than for a relativistic electron. Accurate values for the range of alpha particles in different media can be obtained from the NIST Web site at:

www.physics.nist.gov/PhysRefData/Star/Text/ASTAR-t.html

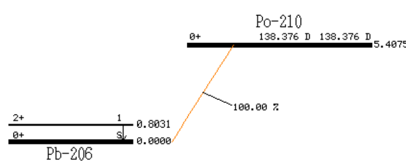

Figure 4. ^{210}Po decay scheme. (From R. G. Helmer and M. A. Lee, *Nuclear Data Sheet* **61**, 93 (1990))

To measure the alpha particle range for ^{210}Po, place the source with the thin window facing upwards in the micrometer fixture. Two different source holders are available to span the useful range of distances. A small magnet is embedded in each source holder to keep the unit fixed on the micrometer screw. Rotate the micrometer spindle to make sure the part is correctly located. Slide the fixture into level 4 of the Geiger tube support assembly, making sure that the support is pushed completely into place. Record the count rate as a function of position for every millimeter from 2 to 13 for the tall source holder and 0 to 13 for the short one. A plot of the count rate will show a very rapid decline when the alpha maximum range is reached. In the region where the sharp decrease occurs, take measurements with ½ mm spacing to get a better estimate of the behavior. To estimate the total particle range, make sure to include the effective thickness of the Geiger counter end window mentioned earlier. Compare with the NIST estimates, assuming an alpha energy of 5.304 MeV. Also, estimate the average energy loss per g/cm^2 of the air absorber. The air path from source to Geiger tube end window is given by:

$$d = 9.06\ mm + x \qquad \text{(tall source support)}$$

or

$$d = 21.25\ mm + x \qquad \text{(short source support)}$$

where x is the reading on the micrometer head (in mm). Finally, correct the counting rate by subtracting the background measured beyond the alpha range cutoff and compute the counting rate per unit of solid angle, $\Delta\Omega$, to compensate for the varying distance from the source to the circular defining aperture at the top of the micrometer assembly. The radius of the circular hole, a, is 3.57 mm and it is located at a distance given by $d' = d - 8.13$ mm where d is defined as above. Thus,

$$\Delta\Omega = 2\pi(1-\cos(\theta)) = 2\pi\left(1-\frac{d'}{\sqrt{d'^2+a^2}}\right)$$

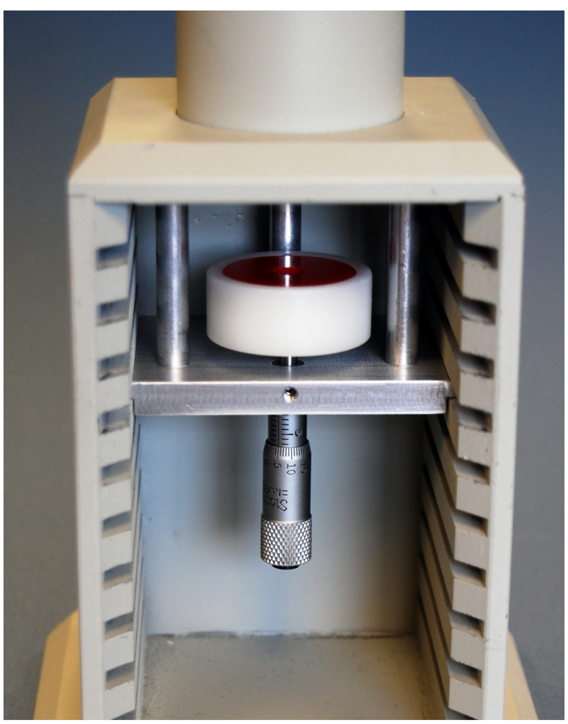

Figure 5. Equipment arrangement for α-particle range measurements.

Figure 6. Equipment arrangement for β-particle range measurements.

Measure the half-life of ^{210}Po: Because nuclear forces are exceedingly short-range, radioactive nuclei decay independently at a fixed rate. This leads to an exponential decrease of intensity with time, usually specified by the *half-life*, $t_{½}$. In one half-life, the intensity drops by a factor of two. The more useful number is the mean life, τ, defined by the exponential decay behavior:

$$I(t) = I_0 \exp(-t/\tau)$$

The two parameters are related by:

$$t_{½} = \log_e(2) \cdot \tau$$

One nuisance of this experiment is that the ^{210}Po isotope decays relatively quickly with a 138-day half-life so that these sources must be replenished once each year. However, it also makes it just possible to observe the effects of decay over a relatively brief period. From your measurements of the α-particle intensity as a function of air path, choose a particular distance well away from the range limit but also not so close that the source is right up against the circular aperture stop. Count the source for long enough to accumulate at least 1% statistics, then repeat the same measurement once each week for the remaining two weeks of the lab. The background is not negligible so make good determinations of these rates as well. Compute the half-life and statistical error from these measurements and compare with the accepted value of 138.376 days.

Measure the range of β^- particles in polyethylene and aluminum: For these measurements, use the ^{90}Sr and ^{204}Tl sources. ^{90}Sr has a slightly complicated decay chain – it first decays by a 546.0 keV β^- to ^{90}Y (yttrium) that decays in turn with a 64-hour half-life by emitting a 2280.1 keV β^- to become a stable ^{90}Zr nucleus. For this experiment, the more energetic beta will be the particle of interest. The ^{204}Tl provides a 763.4 keV β^- along with a ^{204}Pb nucleus. Although the Pb isotope is unstable against alpha decay, the transition lifetime corresponds to a million times the age of the Universe. Note that unlike alpha decay, the emitted beta particles have a continuum of energies up to the maximum allowed by energy conservation since all of these transitions include the creation of an unseen neutrino. Unlike the measurements of the alpha range, a plot of the count rate as a function of absorber thickness will reveal a continuous monotonic decrease until the natural background level is reached. This endpoint should correspond to the maximum beta energy. For each source, do this twice; first, with polyethylene absorbers and second, with aluminum absorbers. Both the polyethylene and aluminum absorbers are distributed in sets of 5 with thicknesses increasing by a factor of two between successive pairs. Thus 31 different thicknesses can be obtained ranging from 0.031″ to 0.969″ for $(CH_2)_n$ and from 0.016″ to 0.484″ for Al. For each isotope, make a semilogarithmic plot of count rate vs. absorber thickness in g/cm^2 for $(CH_2)_n$ and Al and compare the endpoints for the two isotopes. For NIST computations of the range of betas, see:

physics.nist.gov/PhysRefData/Star/Text/ESTAR.html

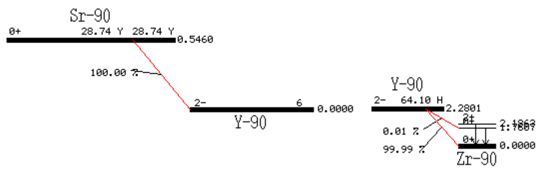

Figure 7. ^{90}Sr - ^{90}Y decay scheme. (From E. Browne, *Nuclear Data Sheet* **82**, 379 (1997))

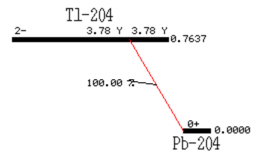

Figure 8. ^{204}Tl decay scheme. (From M. R. Schmorak, *Nuclear Data Sheet* **72**, 409 (1994))

Interpolate the values of the CSDA Range in column 4 for the endpoint energies of ^{90}Y and ^{204}Tl (CSDA = *continuous-slowing-down approximation*). Your measurements will fall below the CSDA values because the electrons scatter in angle as they slow down. The *Projected Range* is the appropriate measure that includes this effect (see O. Gürler, *et al*. for values for C and Al). The betas from ^{90}Y are moderately relativistic with v/c = 0.983. At these velocities, the energy loss rate is close to a minimum beyond which dE/dx increases logarithmically. From your data, estimate the energy loss rate for the β from ^{90}Y and use this value to find the original energy of a muon created in the upper atmosphere that survives the trip to be counted in our classroom. What is the ratio of energy loss rates for these betas relative to the alpha particles from ^{210}Po?

E (MeV)	CSDA range (gm/cm^2)			Projected range (gm/cm^2)	
	C	CH_2	Al	C	Al
0.10	1.602×10^{-2}	1.339×10^{-2}	1.872×10^{-2}	8.250×10^{-3}	6.391×10^{-3}
0.20	5.032×10^{-2}	4.215×10^{-2}	5.804×10^{-2}	2.703×10^{-2}	2.138×10^{-2}
0.30	9.462×10^{-2}	7.924×10^{-2}	0.1083	5.241×10^{-2}	4.210×10^{-2}
0.40	0.1450	0.1213	0.1652	8.230×10^{-2}	6.700×10^{-2}
0.50	0.1993	0.1667	0.2260	0.115	9.496×10^{-2}
0.60	0.2561	0.2142	0.2894	0.151	0.1252
0.70	0.3147	0.2632	0.3545	0.188	0.1571
0.80	0.3746	0.3133	0.4206	0.226	0.1902
0.90	0.4352	0.3641	0.4874	0.265	0.2243
1.00	0.4964	0.4155	0.5546	0.305	0.2591
1.50	0.8062	0.6759	0.8913	0.510	0.4369
2.00	1.1160	0.9375	1.2240	0.716	0.6130
2.50	1.4230	1.1970	1.5500	0.918	0.7815
3.00	1.7270	1.4550	1.8690	1.114	0.9400

Table I. Electron ranges in various materials. CSDA ranges taken from the NIST ESTAR Web site, projected ranges from O. Gürler, *et al*.

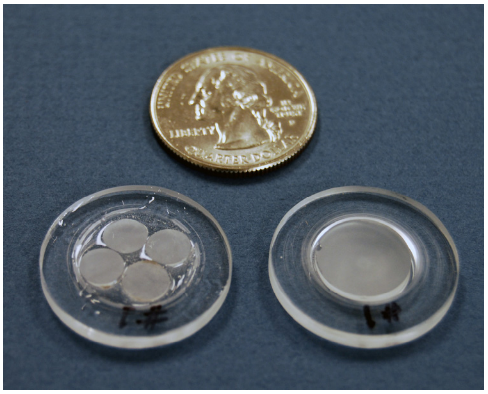

Figure 9. KCl crystals mounted on 1″ diameter acrylic disks.

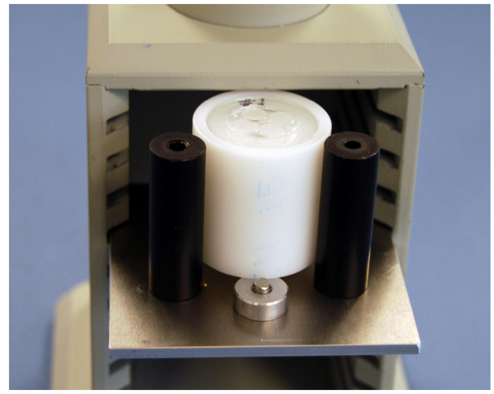

Figure 10. KCl sample on Delrin support ready to be inserted under Geiger counter.

Measure the radioactivity of potassium chloride and extrapolate these values to the human body: There are two common radioactive isotopes in the human body, ^{14}C and ^{40}K. The first of these has a half-life of 5730 years and is continually regenerated by cosmic ray interactions in the upper atmosphere. The β^- from this decay has a maximum energy of 156 keV that is relatively difficult to detect. The ^{40}K has a half-life of 1.277×10^9 years and is left over from the stellar nuclear synthesis processes that formed the rest of the heavy elements on Earth (to astronomers, these are all 'metals'). (^{40}K is an example of an odd-odd nucleus that would prefer to be even-even by transforming to either ^{40}Ca or ^{40}Ar.) The β^- for the decay to ^{40}Ca has a maximum energy of 1.3121 MeV and traverses material with areal densities of the order of 0.3 grams/cm^2 (~1.5 mm).

In this experiment, you will count the β^- rate for two samples of crystalline KCl mounted on 1″ diameter acrylic disks. For one of these disks, the KCl crystal is a single disk, 13 mm in diameter by 2 mm thick; the other contains four disks, each 6 mm diameter and 1 mm thick. By measuring both, you can estimate the β^- attenuation and correct appropriately. The samples should be placed on top of a 1.3″ high Delrin cylindrical support so that they will be as close as possible to the detector (see Figures 9 and 10). (When finished with counting, please return samples to the sealable plastic bags to avoid degradation.) When properly located, the KCl disks are 0.29″ from the Geiger tube window which, in turn, is about 1.1″ in diameter.

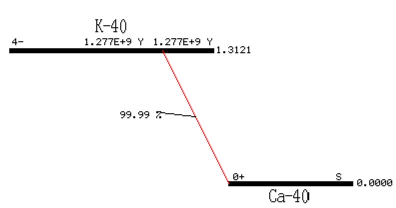

Figure 11. ^{40}K decay scheme. (From P. M. Endt, *Nuclear Physics* **A521**, 1 (1990))

Data from the two different thicknesses allow you to correct for internal particle absorption in the crystals, assuming exponential attenuation. After applying corrections for attenuation, geometric coverage and branching fraction to ^{40}Ca, compute the decay rate per gram of KCl. Compare this value with the rate estimated from the half-life of ^{40}K and its known natural isotopic abundance. Extrapolate to the total amount of potassium in your body, assuming a concentration of 2 grams of potassium per kilogram of tissue. What is your estimated radioactivity in Curies? (1 Ci = 3.7×10^{10} disintegrations per second.) Bananas are rich in potassium, about 0.5 grams a piece. What is the equivalent radioactivity? If you have time, measure the endpoint energy for the ^{40}K betas by finding the thickness of polyethylene or aluminum to stop most particles.

^{40}K $t_{½}$	1.277×10^{9} y
Branching fraction to ^{40}Ca	89.28 %
^{40}K abundance	0.0117 %
K atomic weight	39.0983
Cl atomic weight	35.4527
KCl density	1.987 g/cm^3

Table II. Physical properties of ^{40}K and KCl.

The lifetime of ^{40}K is greater than a billion years. Consider determining this lifetime by measuring the decrease of radioactivity with time as was described earlier for ^{210}Po. For the sake of keeping this problem simple, assume that you have exactly one gram of KCl for a sample. Assume your counter detects every single decay (ie. covers all 4π steradians of solid angle). What is the minimum length of time that you would have to count to determine the lifetime to 10% accuracy? What is the required stability of counting efficiency throughout this period?

Measure the attenuation coefficient for ^{60}Co and ^{137}Cs γ-rays: ^{60}Co predominantly emits two gamma-rays with energies of 1173.237 keV and 1332.501 keV; ^{137}Cs emits a single 661.657 keV photon. Unlike charged particle interactions in which the ionizing particle loses a small fraction of its energy with each atomic collision, gamma-ray interactions are essentially all or nothing propositions. The consequence is that a counting rate vs. absorber thickness curve will be a decaying exponential rather than a step function or linear decrease. Particularly for photons with energies above 500 keV, the absorption length is considerably longer than for alphas or betas.

For this experiment, use the tungsten disk absorbers, placing them in the same holder employed for the ^{40}K measurements performed earlier (see Figures 10, 14 and 15). Starting with the ^{137}Cs source, you will discover that the count rate without any absorber is quite high but drops by a factor of ~15 with the intervention of a small thickness of polyethylene (~0.06″) as shown in Figure 14. The initial high rate is due to the β^- component. Since the number of β^- and γ are initially almost the same, the large rate decrease demonstrates that the Geiger tube is relatively insensitive to gamma-rays, a consequence of the low density of the fill gas.

Plot graphs of the count rate vs. absorber areal density measured in grams per square centimeter and fit to an exponential decay plus a constant background. To obtain the density, weigh each of the disks and divide by the area corresponding to a diameter of

1.25″. Calculate the χ^2 for these fits and find the probability that your results are consistent with your mathematical model of exponential decay plus constant background. (In *Excel*, the relevant function is CHIDIST). What would be the effect of omitting the constant background term? For the photon energies involved, the interaction processes are predominantly Compton scattering and the photoelectric effect. Compare your values for the attenuation coefficients with values obtained from the NIST Web site:

physics.nist.gov/PhysRefData/Xcom/Text/XCOM.html

The values you should compare with are given in the last column: *Total Attenuation – Without Coherent Scattering*. Note that the Web site allows you to specify the specific γ-ray energies of interest.

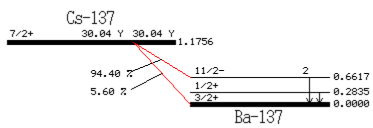

Figure 12. ^{137}Cs decay scheme. (From J. K. Tuli, *Nuclear Data Sheet* **81**, 579 (1997))

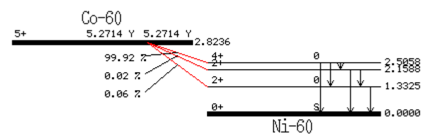

Figure 13. ^{60}Co decay scheme. (From M. M. King, *Nuclear Data Sheet* **69**, 1 (1993))

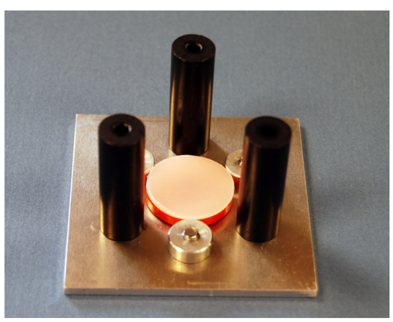

Figure 14. Gamma-ray source with 1/16″ CH_2 absorber.

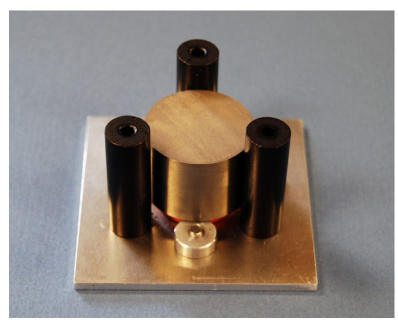

Figure 15. Gamma-ray source with stacked W absorbers.

Overall conclusions: In completing the report for this experiment, summarize the important characteristics of α, β, and γ radiation that were observed. What were the limitations of the Geiger tube detector system and what effects did that have on your results?

References

Glenn F. Knoll**,** *Radiation Detection and Measurement, Third Edition*, J. Wiley (January 5, 2000).

Ernest M. Henley & Alejandro Garcia, *Subatomic Physics*, Third Edition, World Scientific (2007).

Kenneth S. Krane, *Introduction to Nuclear Physics*, J. Wiley (October 22, 1987).

Konrad Kleinknecht, *Detectors for Particle Radiation, Second Edition*, Cambridge University Press (February 15, 2001).

E. C. Anderson and W. H. Langham, *Average Potassium Concentration of the Human Body as a Function of Age*, Science **130**, 713-714 (1959).

Orhan Gürler, Sezai Yalçin, Ali Gültekin and Gökay Kaynak, *Calculation of Projected Range of Electrons in Some Elemental Solids and Gases*, Japanese Journal of Applied Physics **44**, 8630–8634 (2005). www.jstage.jst.go.jp/article/jjap/44_Part1/12/8630/_pdf

Nuclear Data Evaluation Laboratory, Korea Atomic Energy Research Institute, *Table of Nuclides: Mass, Half life, Decay scheme, Cross sections*, http://atom.kaeri.re.kr/