Measurement of Galactic Rotation Curve

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Objective:

The 21-cm line produced by neutral hydrogen in interstellar space provides radio astronomers with a very useful probe for studying the differential rotation of spiral galaxies. By observing hydrogen lines at different points along the Galactic plane one can show that the angular velocity increases as you look at points closer to the Galactic center. The purpose of this experiment is to create a rotational curve for the Milky Way Galaxy using 21-cm spectral lines observed with a small radio telescope.

The sample observations for this experiment will be made using the small radio telescope located at the Haystack Observatory. The rotational curve will be created by plotting the maximum velocity observed along each line of sight versus the distance of this point from the Galactic center.

Introduction:

The 21-cm line of neutral hydrogen

Hydrogen is the most abundant element in the cosmos; it makes up 80% of the universe’s mass. Therefore, it is no surprise that one of the most significant spectral lines in radio astronomy is the 21-cm hydrogen line.

In interstellar space, gas is extremely cold. Therefore, hydrogen atoms in the interstellar medium are at such low temperatures (~100 K) that they are in the ground electronic state. This means that the electron is as close to the nucleus as it can get, and it has the lowest allowed energy. Radio spectral lines arise from changes between one energy level to another.

A neutral hydrogen atom consists of one proton and one electron, in orbit around the nucleus. Both the proton and the electron spin about their individual axes, but they do not spin in just one direction. They can spin in the same direction (parallel) or in opposite directions (anti-parallel). The energy carried by the atom in the parallel spin is greater than the energy it has in the anti-parallel spin. Therefore, when the spin state flips from parallel to anti parallel, energy (in the form of a low energy photon) is emitted at a radio wavelength of 21-cm. This 21-cm radio spectral line corresponds to a frequency of 1.420 GHz.

The first person to predict this 21-cm line for neutral hydrogen was H. C. van de Hulst in 1944. However, it was not until 1951 that a Harvard team created the necessary equipment, and the first detection of this spectral line was made.

One reason this discovery is so significant is because hydrogen radiation is not impeded by interstellar dust. Optical observations of the Galaxy are limited due to the interstellar dust, which does not allow the penetration of light waves. However, this problem does not arise when making radio measurements of the HI region. Radiation from this region can be detected anywhere in our Galaxy.
Measurements of the HI region of the Galaxy can be used in various calculations. For example, observations of the 21-cm line can be used to create the rotation curve for our Milky Way Galaxy. If hydrogen atoms are distributed uniformly throughout the Galaxy, a 21-cm line will be detected from all points along the line of sight of our telescope. The only difference will be that all of these spectra will have different Doppler shifts. Once the rotation curve for the Galaxy is known, it can be used to find the distances to various objects. By knowing the Doppler shift of a body, its angular velocity can be calculated. Combining this angular velocity and the plot of the rotation curve, the distance to a certain object can be inferred. Using measurements of the HI region, the mass of the Galaxy can also be determined.

Procedure:

Use the SRT to observe the H-line spectrum at several points in the plane of the Galaxy. A sample command file is as follows:

```
LST:1800:00:00  / wait until 1800 UST when Galactic center is above
   / telescope limits
   : 1420.4 50  / set radiometer center frequency and a number of frequency bins
   : azel 180 45  / point south at 45 degrees
   : calibrate   / calibrate radiometer
   : Galactic 0 0  / move telescope to Galactic center
   : record g00.rad/ start data file
   : 600  / take data for 600 seconds
   : roff  / turn off data recording
   : Galactic 10 0 / move telescope to Galactic longitude of 10 degrees
   : record g10.rad
   : repeat for longitudes to 90 degrees
```

This suggested observing schedule provides spectra for 10 points in the Galactic plane. Plot the spectra and read off the velocity of the most positive velocity emission relative to the local standard of rest (i.e. after removal of the Doppler shift due to the local motion of the Sun and the Earth). This can be accomplished by running the java program “splot” on the data files, doing your own analysis on the data files or by being present during the observations to make notes or capturing the screen.

The velocity $V$ of a frequency channel is given

$$V = \frac{(1420.406-f) \cdot V_c}{1420.406 - V_{lsr}} \text{ km/s}$$

Where $V_c = \text{velocity of light 299790 km/s}$

$V_{lsr} = \text{velocity of observer relative to local standard of rest}$

$f = \text{frequency of channel} = f_{L.O.} + 0.04$

The real time control program and splot both contain a routine which calculates the $V_{lsr}$ and splot labels the spectrum axis with velocity relative to the local standard of rest.
Sample results:

<table>
<thead>
<tr>
<th>Galactic longitude (deg)</th>
<th>Freq. (MHz)</th>
<th>max velocity km/s</th>
<th>estimated error km/s</th>
<th>tangential distance kpc</th>
<th>( \omega, R ) km/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1419.60</td>
<td>150</td>
<td>-</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>1419.50</td>
<td>180</td>
<td>-</td>
<td>1.5</td>
<td>39</td>
</tr>
<tr>
<td>20</td>
<td>1419.70</td>
<td>130</td>
<td>10</td>
<td>2.9</td>
<td>75</td>
</tr>
<tr>
<td>30</td>
<td>1419.82</td>
<td>120</td>
<td>10</td>
<td>4.2</td>
<td>109</td>
</tr>
<tr>
<td>40</td>
<td>1419.98</td>
<td>95</td>
<td>10</td>
<td>5.5</td>
<td>142</td>
</tr>
<tr>
<td>50</td>
<td>1420.10</td>
<td>70</td>
<td>10</td>
<td>6.5</td>
<td>168</td>
</tr>
<tr>
<td>60</td>
<td>1420.18</td>
<td>60</td>
<td>10</td>
<td>7.4</td>
<td>192</td>
</tr>
<tr>
<td>70</td>
<td>1420.34</td>
<td>30</td>
<td>10</td>
<td>8.0</td>
<td>207</td>
</tr>
<tr>
<td>80</td>
<td>NOT OBSERVED</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>90</td>
<td>1420.42</td>
<td>10</td>
<td>10</td>
<td>8.5</td>
<td>220</td>
</tr>
</tbody>
</table>

Sample spectrum:

![Sample spectrum](chart.png)
Theory:

Model of the Galactic rotation

We model the Galactic motion as circular motion with monotonically decreasing angular rate with distance from the center. For example if the mass of the Galaxy is all at the center the angular velocity at distance \( R \) is given by

\[
\omega = M^{\frac{1}{2}} G^{\frac{1}{2}} R^{-\frac{3}{2}}
\]

where
- \( M = \) central mass kg
- \( G = \) gravitational constant \( 6.67 \times 10^{-11} \text{Nm}^2\text{kg}^{-2} \)
- \( R = \) distance from central mass m

If we are looking through the Galaxy at an angle \( \gamma \) from the center the velocity of the gas at radius \( R \) projected along the line of site minus the velocity of the sun projected on the same line is

\[
V = \omega R \sin \delta - \omega_0 R_0 \sin \gamma
\]  

(1)

as illustrated in figure below

where
- \( \omega = \) angular velocity at distance \( R \)
- \( \omega_0 = \) angular velocity at a distance \( R_0 \)
- \( R_0 = \) distance to the Galactic center
- \( \gamma = \) Galactic longitude
Using trigonometric identity

\[ \sin \delta = R_0 \sin \gamma / R \] (2)

and substituting into equation (1)

\[ V = (\omega - \omega_0) R_0 \sin \gamma \] (3)

(which is equation 8-51 in Kraus)

The maximum velocity occurs where the line of site is tangential to the circular motion in which case

\[ R = \sin \gamma R_0 \] (4)

And hence \[ V_{\text{max}} = \omega R = V_{\text{max}} (R) + \omega_0 R \] (5)

The Sun’s rotational velocity \( \omega_0 R_0 \) and distance to the Galactic center can be taken from other measurements to have values of

\[ \omega_0 R_0 = 220 \text{ km/s} \]
\[ R_0 = 8.5 \text{ kpc} = 2.6 \times 10^{17} \text{ km} \]

**Data analysis:**

\[ V_{\text{max, observed}} (R) + \omega_0 R \] (6)

is plotted using the values of \( V_{\text{max, observed}} \) and the calculated values of \( \omega_0 R \) from table of sample results:

**Interpretation of results:**
The rotational velocity doesn’t depend strongly on the distance from the center at least for $R > 3 \, \text{kpc}$. [Measurements close to the Galactic center are difficult and the integration needs to be sufficient to reduce the noise so that the maximum velocity edge of the emission can be determined.] If the motion was entirely Keplerian (i.e. classical circular orbits around a central mass) the rotational velocity would decrease with a $R^{-\frac{1}{2}}$ dependence. The lack of decline with distance suggests that not all the mass is in the center of the Galaxy.

**Exercises for the student:**

1. What Galactic mass distribution might fit your data?

2. On the basis of Keplerian motion what is an approximate estimate of the total mass of the Galaxy in solar masses ($M_{\text{sun}} \approx 2 \times 10^{30} \, \text{kg}$)?

3. Include the Galactic center in your observations. What can you conclude from the observed spectrum?

4. Include observations at negative Galactic longitude and longitudes greater than 90.

5. Observe the H-line spectrum at longitude 180 degrees. What would you expect to observe on the basis of the simple model of circular motion? What would the H-line spectra look like if the Galaxy rotated as a rigid body?

6. Observe the H-line several points crossing the Galactic plane. Is the H-emission from a thin disk? Or can you resolve the “thickness” of the plane given the resolution of the SRT.

7. How do you explain the sudden transition from the lack of velocity spread in the H-line at Galactic longitude less than 20 degrees to a double peaked spectrum at longitudes greater than 20 degrees?

8. How can you use your Galactic velocity dependence with distance to map the distribution of hydrogen?

The material in the introduction was provided by Maria Stefanis, Mount Holyoke and Larry Kimball, University of Massachusetts Lowell.