Analyzing the retarding potential curves for the photoelectric effect

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The general behavior of the retarding potential curves for the photoelectric effect makes precision measurements of Planck's constant difficult. The problem is best illustrated by a typical graph of the photocurrent vs. retarding voltage as shown in Figure 1 below.

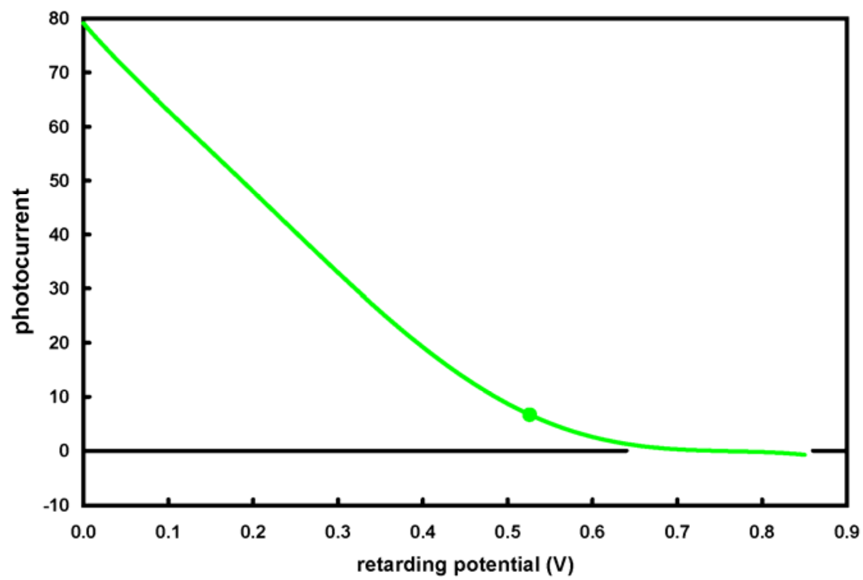

Figure 1. The photocurrent vs. retarding voltage response to green (532 nm) light. The large green dot designates the point of maximum curvature.

The information that we need to find is the applied voltage that causes the photocurrent to go to zero. Unfortunately, the slope of the curve decreases drastically in this region, making the intercept difficult to resolve. This is yet more complicated by a reverse current from electrons ejected by stray light hitting the phototube collector electrode as well as the uncertainty due to the poorly constrained functional behavior of the photoelectron momentum spectrum. The recommendations in this note do not solve these problems completely but they minimize errors due to ambiguous data reduction procedures.

The major task is to find the point of maximum curvature in the photocurrent curve, shown as the large bright dot in Figure 1. In principle, this might be done by computing two stages of finite differences to estimate the second derivative. In practice, even for good data, this is a very noisy process that leads to poorly reproducible results.

The alternative is to smooth the data by fitting to simple analytic curves which emphasizes the global behavior of the data, smoothing out local noise. The curvature of the fitted curve can then be extracted by the usual methods of calculus. For the problem at hand, the following functional form is suggested:

$$y_i = a_0 + a_1 \cdot x_i + a_2 \cdot x_i^2 + a_3 \cdot x_i^3 + a_4 \cdot x_i^4 + a_5 \cdot x_i^5$$

The coefficients, $\{a_j\}$, should be computed over the entire data set using the linear least squares procedure outlined in my lecture notes. Since the variance of the data is not well-known, one should make the assumption that it is uniform over all data. The curvature of the photocurrent curve is then predicted by:

$$d^2y/dx^2 = 2a_2 + 6a_3 \cdot x + 12a_4 \cdot x^2 + 20a_5 \cdot x^3$$

The curvature maximum can be found by finding the value of x that makes d^3y/dx^3 equal to zero. Note that this later equation is quadratic so that a closed form solution exists. Thus, the location of the maximum curvature can be computed as depicted in Figure 1.

If the position of the curvature maximum alone is used as a proxy for the photocurrent cutoff voltage, a significant systematic error will be incurred because curves taken with different wavelengths of light will show sharper or broader curvatures and thus different increments in voltage before all the electrons are fully repelled. A procedure for dealing with this is depicted in Figure 2 below.

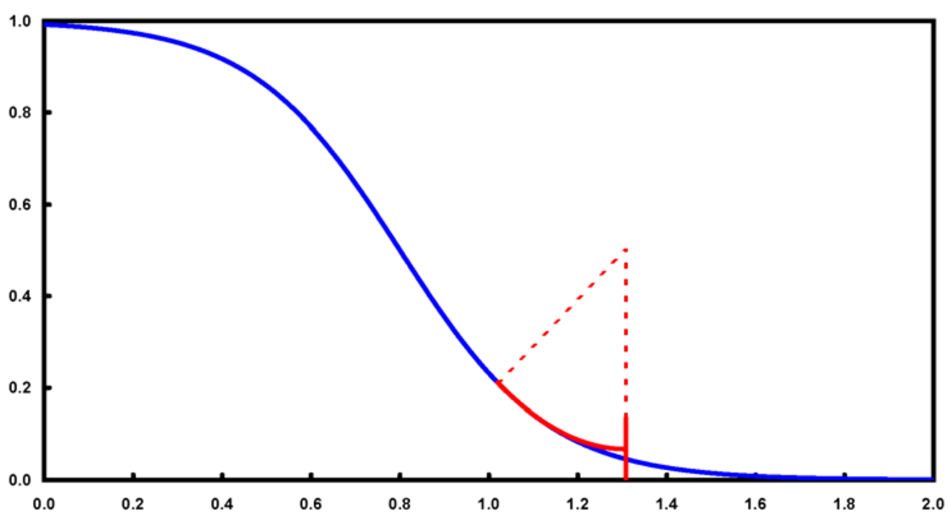

Figure 2. Illustrating the use of the first and second derivatives to estimate the zero-crossing point for the S-shaped curve (blue). The solid red line is a quadratic curve that matches the S-curve at the point of maximum curvature. The end point depicted is sensitive to both the position of the maximum point of curvature and the magnitude.

To compensate for different values of the maximum curvature, we ask the following question: how far do we have to move to the right before the slope of the curve would be zero? That offset is given by:

$$\delta x = - dy/dx \,/\, d^2y/dx^2$$

The addition of δx to the point of maximum curvature thus gives a reasonably unbiased estimate of the photocurrent cutoff voltage. Without a physically motivated model of the photoelectron spectrum, this is probably about as well as one can do.

The statistical error of this procedure can be estimated by the techniques discussed in class although the expressions are messy enough that resort to *Mathematica* or a similar symbolic algebra program is almost essential. The variance of the data can be crudely estimated by scaling the χ^2 value to the number of degrees of freedom given by the number of data points and the number of parameter constraints. The dependence of the cutoff voltage is quite complicated so it's best to import the fitted parameter values to *Mathematica* and let it find the values for the appropriate derivatives. These can be exported back to Excel or whatever package you are using to compute the cutoff variance. It is also possible to investigate whether the choice of analytic functions affects your result. As long as there are no large systematic deviations between data and fitted curve, this is not likely to be a problem.

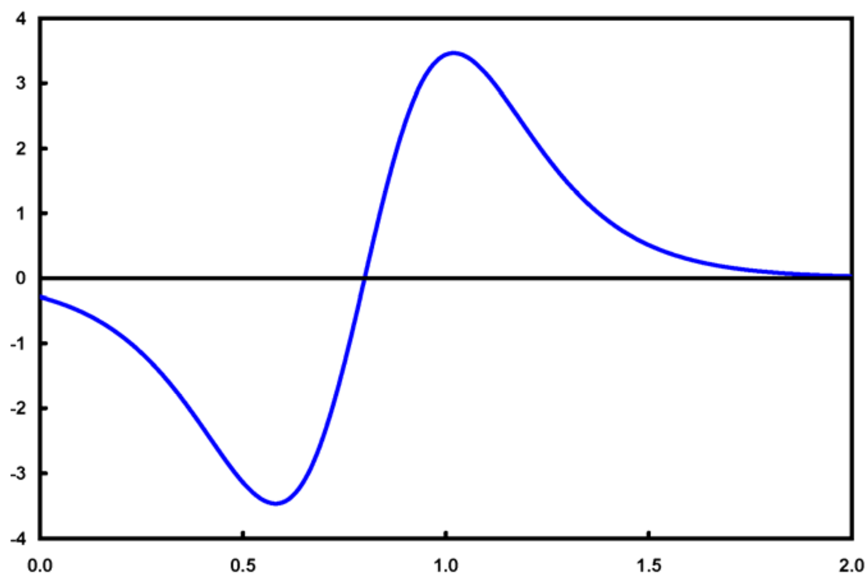

Figure 3. The second derivative of the curve shown in Figure 2. Note the peak near x = 1.0 which corresponds to the point of maximum curvature.