

Electron-Positron Pair Production

Carl W. Akerlof
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Introduction

An essentially unique aspect of physics is the almost miraculous ability of pure theory to make predictions of phenomena never seen before. One such example was the discovery of the positron based on unanticipated solutions of the Dirac relativistic wave equation^{1, 2}. Dirac first suggested that the positive particle in his equations might represent the proton³ but soon realized that this would not work⁴. Fortunately, Carl Anderson soon discovered the positron in cloud chamber photographs^{5, 6} and the uncanny genius of Dirac's work became a legend. (See a review⁷ by J. H. Hubbell for more detail.)

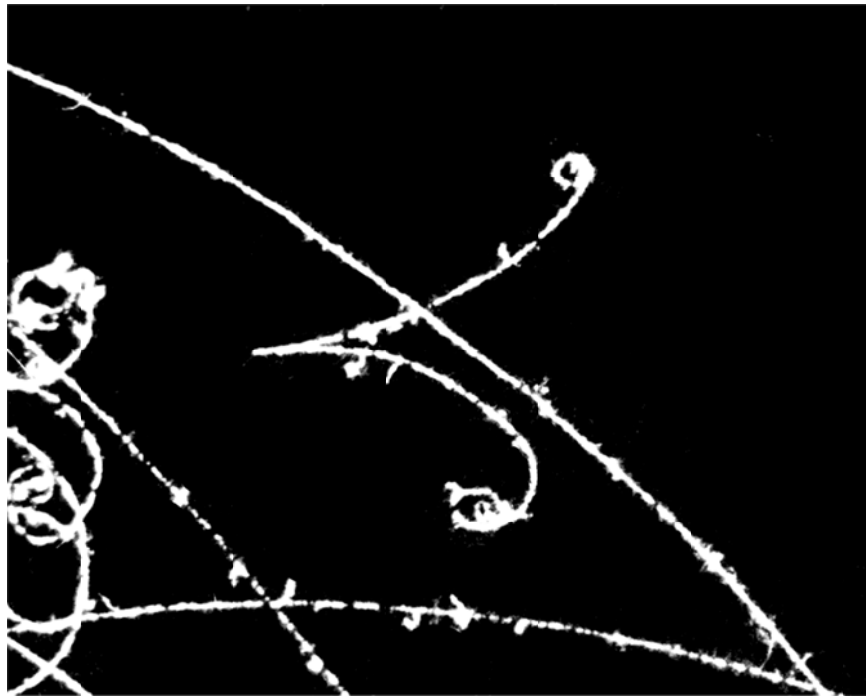


Figure 1. Portion of a bubble chamber photograph showing the creation of an electron-positron pair in a magnetic field. (Image courtesy of J. C. Vandervelde.)

This mode of prediction hopefully followed by discovery very much persists to the present moment. Dirac's paper⁴ acknowledging that the proton was not the negative energy solution of his relativistic quantum mechanics also showed that if magnetic monopoles existed, they would be quantized with an effective charge proportional to e/α where α is the fine structure constant. About 30 years ago, that created a frenzy of experimental activity following papers^{8, 9} by t'Hooft and Preskill indicating that such objects were natural consequences of Unified Gauge

Theories and could have been produced in the early Universe. Sadly, if there are magnetic monopoles, cosmic inflation has killed their density to levels that are undetectable by the most heroic efforts. At this moment, the physics community is waiting to see if another prediction will be confirmed, the existence of the Higgs boson¹⁰. Following tantalizing hints from CERN and Fermilab, we should know the answer to this question before of the end of the calendar year 2012.

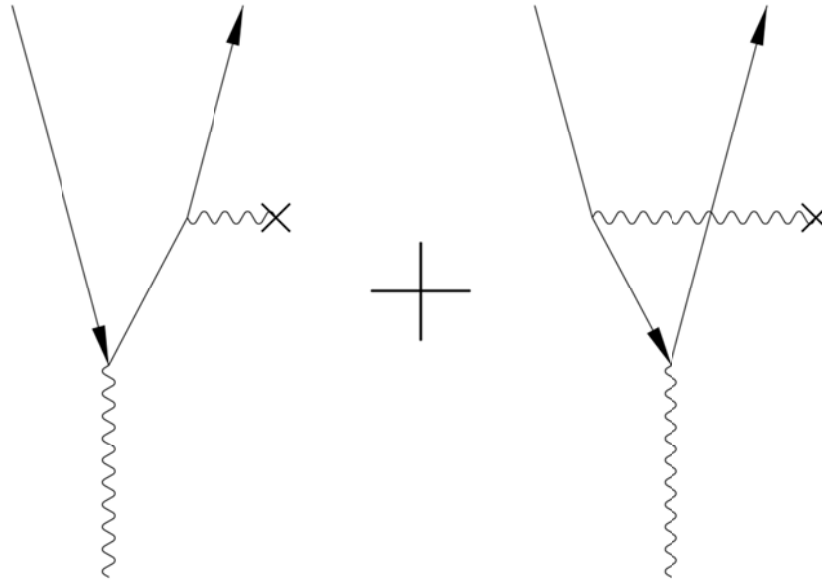


Figure 2. Lowest order Feynman diagrams for e^-e^+ pair production in the field of a nucleus.

Some theoretical considerations

The basic lowest-order Feynman diagrams that describe pair production are shown in Figure 2. Two photons are required to conserve energy and momentum. In the drawing above, one of those photons is virtual and is supplied by the electrostatic field of an atomic nucleus. Thus the amplitude scales like the atomic number, Z , and the cross-section as Z^2 . Since the final state contains both an electron and a positron, the absolute minimum incident photon energy for this process is $2m_e c^2$. As you might expect, the cross-section rises rapidly above threshold. It reaches a plateau at energies above 1 GeV with an asymptotic value given by:

$$\sigma_{e^+e^-} = \frac{28}{9} \frac{Z^2 \alpha^3 (\hbar c)^2}{(m_e c^2)^2} \left[\log \left(\frac{183}{Z^{1/3}} \right) - \frac{2}{7} \right]$$

Problem 1: Calculate the threshold photon energy for pair production from a free electron.

Problem 2: Compute the threshold photon energy for pair production from interactions with the photons that constitute the cosmic microwave background radiation (CMBR). The CMBR black-body temperature is 2.725° K. This process limits the transparency of the Universe for high energy photons.

Description of experiment

There are three major components of this experiment:

- 1) ^{226}Ra radioactive source to provide > 1 MeV photons.
- 2) Set of six converter plates fabricated from carbon, aluminum, copper, tin, tungsten and lead.
- 3) Germanium solid state radiation detector.

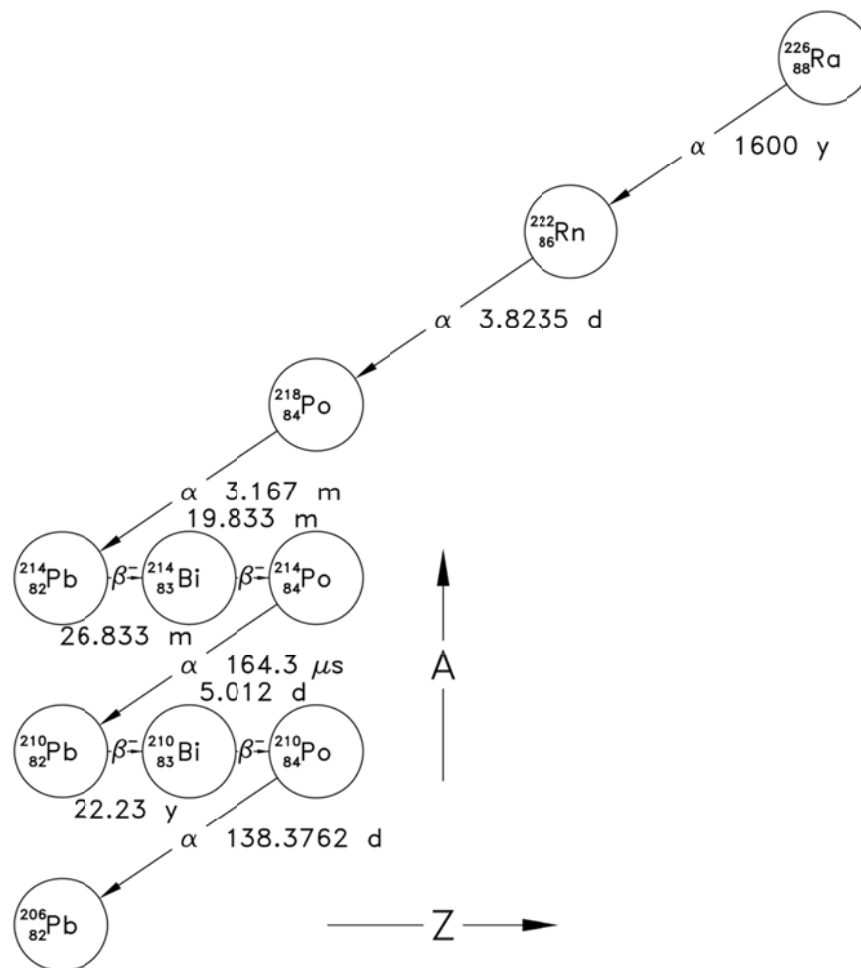


Figure 3. Decay chain for the ^{226}Ra radioisotope.

The radium source has a long and complicated decay chain as shown in Figure 3. These transitions emit alpha, beta and gamma rays over a broad range of energies. Since ^{226}Ra is directly descended via alpha and beta decay from ^{238}U , all of the gamma-ray lines can be detected at a low level in the natural radiation background. See <http://atom.kaeri.re.kr/> for detailed diagrams of the decay processes for each isotope in the chain.

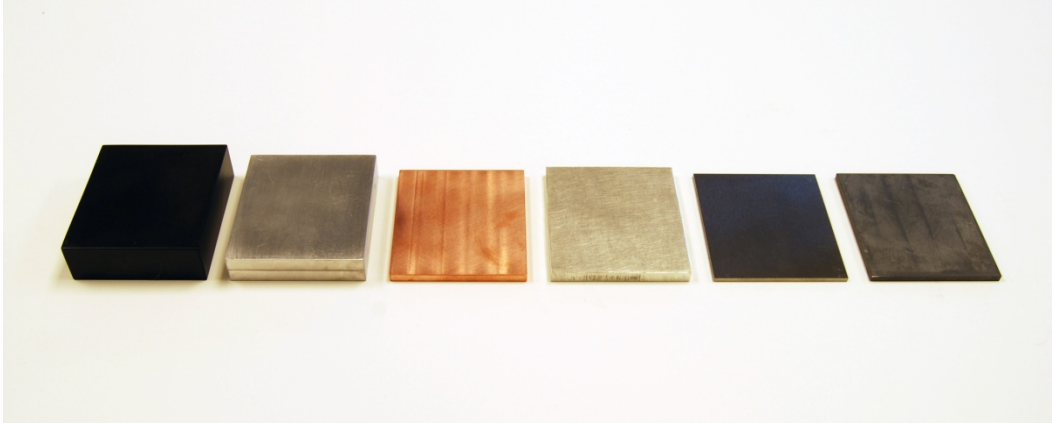


Figure 4. Six converter plates available for this experiment, From left to right, carbon, aluminum, copper, tin, tungsten & lead.

The main focus of this experiment is the measurement of the pair production yield as a function of the atomic number of the target material. Six converter plates are provided with the physical characteristics listed in Table I below.

	Z	A	X (cm)	Y (cm)	Z (cm)	M (g)
C	6	12.0107	7.620	8.635	2.547	294.0
Al	13	26.981538	7.605	8.637	1.750	311.3
Cu	29	63.546	7.623	8.638	0.537	315.7
Sn	50	118.71	7.785	8.730	0.730	362.4
W	74	183.84	7.634	8.652	0.275	307.7
Pb	82	207.2	7.635	8.640	0.394	291.6

Table I. Physical dimensions of pair production converter plates.

To compare the relative effects of different atomic number, you will need to divide the number of detected pair events by the number of target nuclei per unit area. From the numbers provided above, that can be determined from the formula:

$$\# \text{ nuclei/area} = \frac{N_A \cdot M}{A \cdot X \cdot Y}$$

where N_A is Avogadro's Number.

The key component of this experiment is the high purity Ge solid state detector which is efficient for detecting gamma-rays while providing excellent energy resolution. The basic geometry of the active volume is a cylinder 52.7 mm in diameter by 54.1 mm long. This device should be operated with a positive potential of 3000 v and must be maintained at cryogenic temperatures with liquid nitrogen. **Make sure the detector is adequately cooled before switching on the detector bias voltage.** A diagram and photograph of the detector and its housing are shown in Figure 5 below.

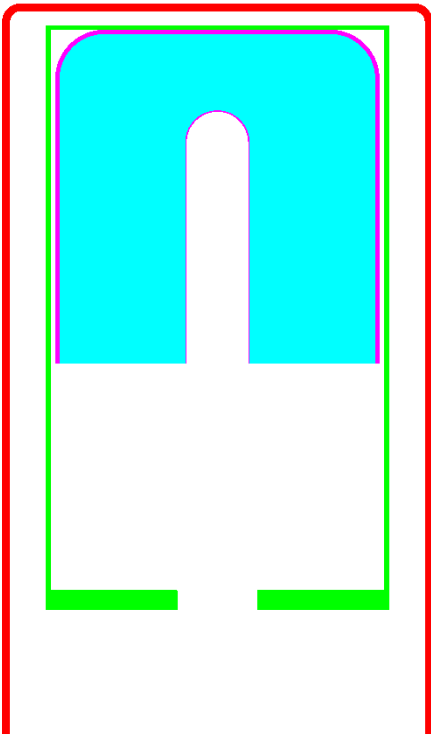


Figure 5a. Cross-section of the high purity Ge detector. The germanium is shown in cyan with electrodes drawn in magenta. The green lines indicate the internal liquid nitrogen shield and the outer vacuum housing is drawn in red.

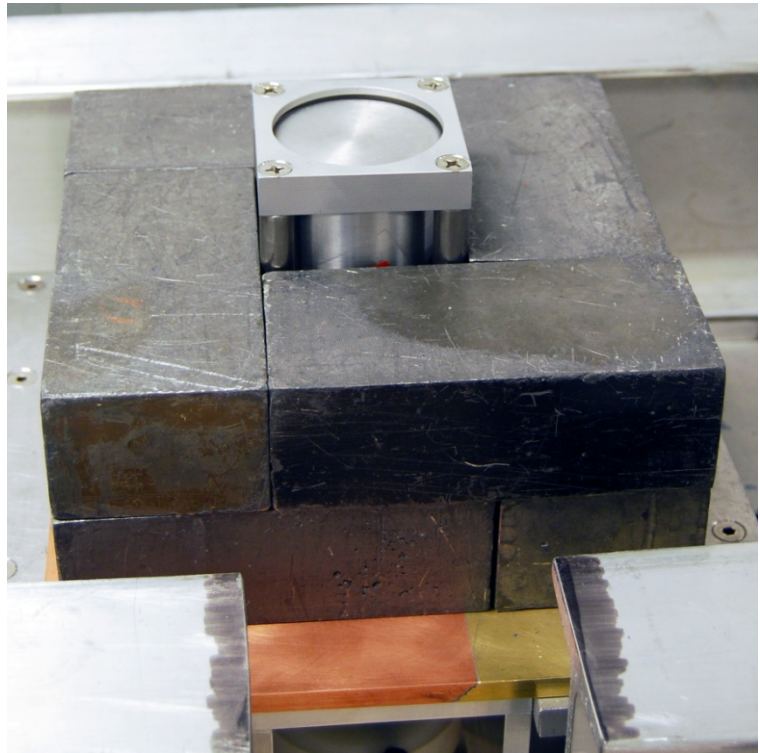


Figure 5b. Photograph of the high purity Ge detector encased in its vacuum housing and surrounded by lead bricks and bracket to support radioactive sources.

The Ge detector is connected by a coaxial cable to the rear panel of the NIM HV bias supply. The detector analog signal output is connected via coax to the input of a linear amplifier module as shown in Figure 6. The output of this unit is, in turn, fed to an MCA (multi-channel analyzer) which is read out by the desktop PC nearby. The only electronic controls that you should possibly change are the linear amplifier gains. For this experiment, the COARSE and FINE gains should be adjusted to put the 1460.830 KeV γ -ray that follows the $^{40}\text{K} \rightarrow ^{40}\text{Ar}$ electron capture into an MCA channel close to 2297, assuming a total of 4096 (see Figure 8). The ^{40}K source is a small bottle of “NoSalt” salt which was purchased at Kroger’s.

The Amptek MCA8000A ‘POCKET MCA’ is accessed via the desktop computer standing nearby. Log on to the PC using the username, umroot\phys-advlab2. The password is prominently posted in the classroom. The data acquisition program for the MCA can be initiated by double clicking on the desktop icon shown below:



ADMCA

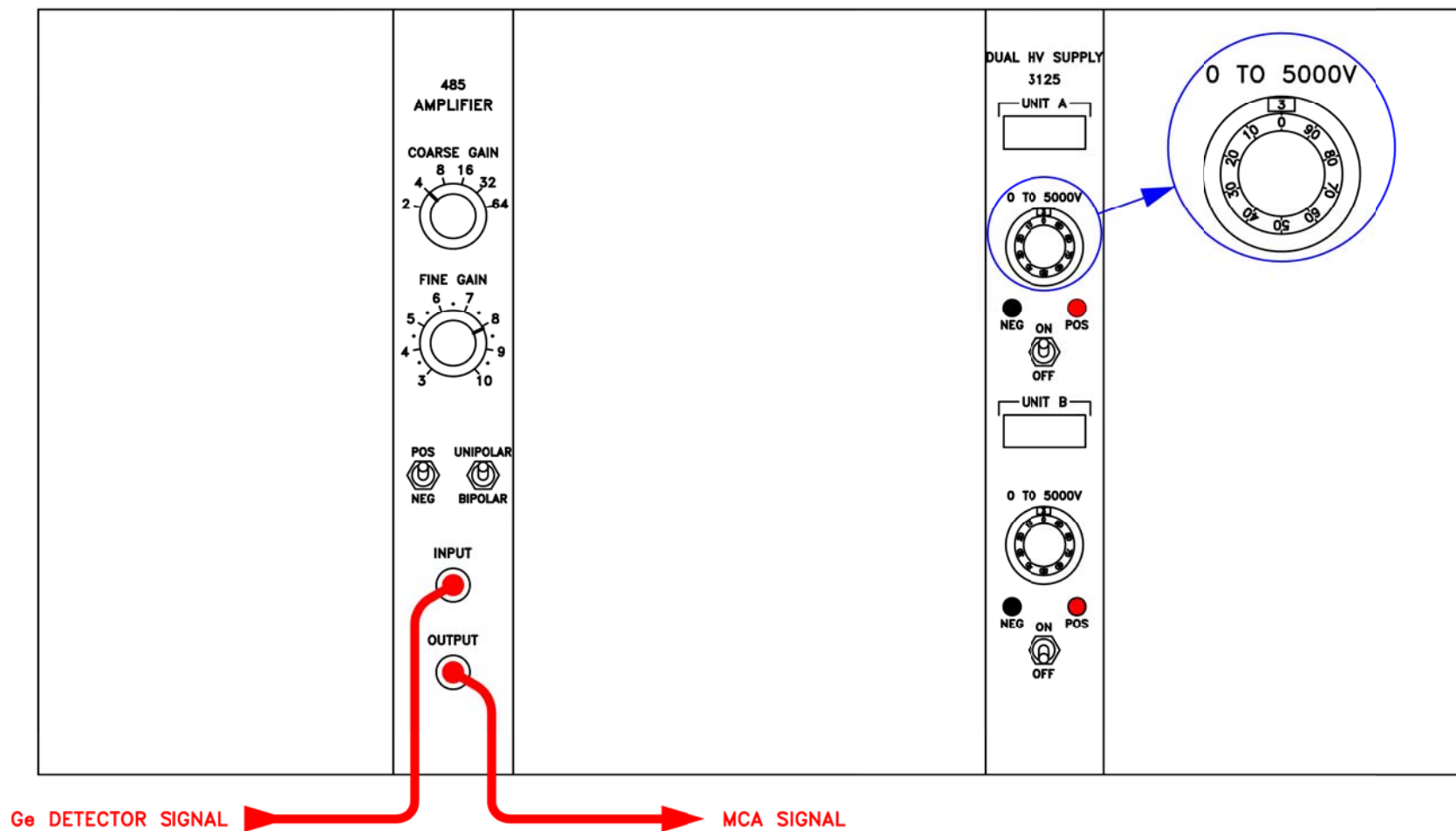


Figure 6. NIM electronics for gamma-ray spectroscopy.

After entering the ADMCA program, you will immediately see the command window shown in Figure 7. Select the *Connect* button and the window will disappear.

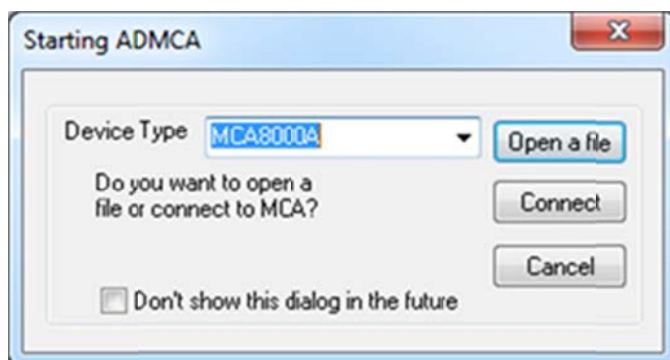


Figure 7. ADMCA initialization window.

There are a variety of pull-down menus for controlling this program. The only ones of significant interest are:

File

Copy Spectrum Data	Ctrl+R	Use this to transfer data to EXCEL.
Capture Plot	Ctrl+C	In case you want to copy and paste to Word.

MCA

Acquisition Setup...	F9	Activate this to control count duration.
Start Acquisition	F3	Start acquisition.
Delete Data and Reset Time	A	Delete data and reset time.

Display

Scale	L ►	Select Lin, Log or Sqrt vertical scaling.
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On the lower right-hand corner of the screen is a set of up, down, left and right arrows. Click the up or down arrows to change the vertical scaling of the spectrum display.

The output of this program will be gamma-ray spectra taken under a variety of conditions. Since there are more than 4000 data channels for each measurement, that's a lot of numbers to play with. You are strongly urged to capture all of the spectral data for every measurement in a common EXCEL spreadsheet formatted to look like Figure 9. This means activating the *Copy Spectrum Data* button under the **File** menu followed by a *Paste* operation into an EXCEL cell somewhere on line 10. Paste successive data sets four columns apart and add information by hand on lines 6 and 8 to describe each data set and its corresponding integration time. When you have accumulated all the data needed for this experiment, open your EXCEL file and perform a *Save As* operation to a tab-delimited text file named *gamma_data.txt*. Move or copy this file to the directory, *C:\Users\phys-advlab2\Documents\gamma_data*, on the data acquisition computer, *PHYS-ADVLAB157*. For convenience, there is a short-cut labeled *gamma data* on the *Desktop* that will pop up the directory immediately. That data file can now be

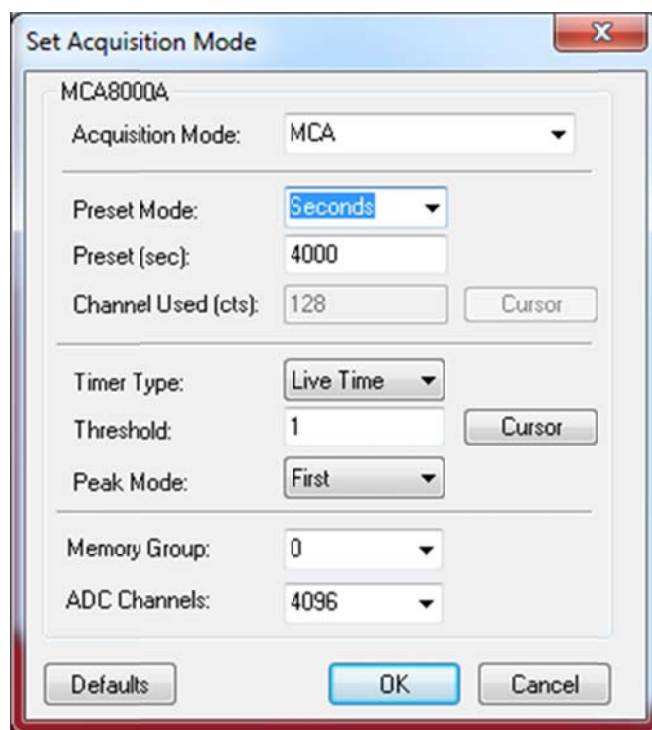


Figure 8. Set Acquisition Mode window.

accessed by an IDL program, *gamma_peaks*, that will do its best to find all the spectral peaks and compute their integrated number of counts. This program is activated by double-clicking the *gamma_peaks* icon on the screen desktop which invokes the IDL code. If the program functions properly, a very large number of graphs will be created before your very eyes. The most useful file created by this operation is *gamma_peaks.txt* (in the *gamma_data* sub-directory) which contains a list of ten parameters for each peak in every data set (see list below). The peak location is given in terms of MCA channel number; it is your job to correlate this with the actual gamma-ray energy. The *gamma_peaks.txt* file is tab-delimited and can be easily read into EXCEL or any other program to extract whatever information is required. You will also find Encapsulated Postscript plot files in the *gamma_data* sub-directory showing the spectrum for each data set with the spectral peaks enumerated corresponding to the *gamma_peaks.txt* output. See Appendix A for more details of the IDL program.

Parameter	Description
1	Data set number
2	Spectral peak number
3	Channel peak
4	Estimated number of integrated counts
5	Estimated integrated count error
6	Line shape λ value
7	Line shape σ value
8	Background counts per channel
9	Lower channel range limit
10	Upper channel range limit

pair_prod_data.xlsx - Microsoft Excel

FileHomeInsertPage LayoutFormulasDataReviewView

ThemesColorsFontsEffectsThemes

MarginsOrientationSizePrint AreaBreaksBackgroundPrint Titles

Page Setup

Width: AutomaticHeight: AutomaticScale: 100%Scale to Fit

GridlinesViewPrintSheet Options

Bring ForwardSend BackwardSelection Pane

AlignGroupRotateArrange

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	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	e ⁺ e ⁻ pair production data														
2															
3	Carl W. Akerlof														
4	16-Feb-12														
5															
6	Ra-226 source				background				Ra-226, 0.253" W				Ra-226, 0.401" Pb		
7															
8	2000 s				2000 s				2000 s				2000 s		
9															
10	Filename: live_data.mca				Filename: live_data.mca				Filename: live_data.mca				Filename: live_data.mca		
11	Description:				Description:				Description:				Description:		
12	TimeStamp: 02/16/2012 14:52:48				TimeStamp: 02/16/2012 16:41:09				TimeStamp: 02/16/2012 15:48:46				TimeStamp: 02/16/2012 14:52:48		
13															
14	Spectral Data Information				Spectral Data Information				Spectral Data Information				Spectral Data Information		
15		Channel	Data			Channel	Data			Channel	Data			Channel	Data
16		0	35511			0	25482			0	29783			0	2774
17		1	18907			1	5055			1	9424			1	810
18		2	11226			2	1066			2	3615			2	315
19		3	9568			3	738			3	2873			3	236
20		4	8295			4	594			4	2516			4	206
21		5	7714			5	515			5	2325			5	183
22		6	7011			6	462			6	2082			6	169
23		7	6479			7	437			7	2005			7	169
24		8	6076			8	414			8	1781			8	154
25		9	5965			9	379			9	1801			9	145
26		10	5645			10	376			10	1650			10	145

Sheet1Sheet2Sheet3

Ready

100%

Figure 9. EXCEL spreadsheet format for pair production data. Store MCA data starting on line 10. Text identification should be on line 6 and integration time on line 8.

Naively one would expect that each individual gamma-ray line would be a symmetric Gaussian shape. In fact, probably due to inefficiencies in charge collection in the Ge detector, the waveform is decidedly skewed with a much longer tail on the low energy side of the peak. A mathematical model for this behavior is shown graphically in Figure 10. The curve is the concatenation of a simple exponential on the left (in red) and a Gaussian on the right (in blue). The two curves are joined slightly to the left of the maximum value with continuous values and first derivatives at the breakpoint, $x_b = -\sigma^2/\lambda$. This model is used in the *pair_prod_peaks* program.

$$f(x) = \begin{cases} ae^{\frac{x}{\lambda} + \frac{\sigma^2}{2\lambda^2}} & ; \quad x \leq -\frac{\sigma^2}{\lambda} \\ ae^{-\frac{x^2}{2\sigma^2}} & ; \quad x > -\frac{\sigma^2}{\lambda} \end{cases}$$

$$\int_{-\infty}^{\infty} f(x) dx = a \left[\lambda e^{-\frac{\sigma^2}{2\lambda^2}} + \sqrt{\frac{\pi}{2}} \sigma \left(1 + \operatorname{erf} \left(\frac{\sigma}{\sqrt{2}\lambda} \right) \right) \right]$$

$$\operatorname{erf}(u) = \frac{2}{\sqrt{\pi}} \int_0^u e^{-t^2} dt$$

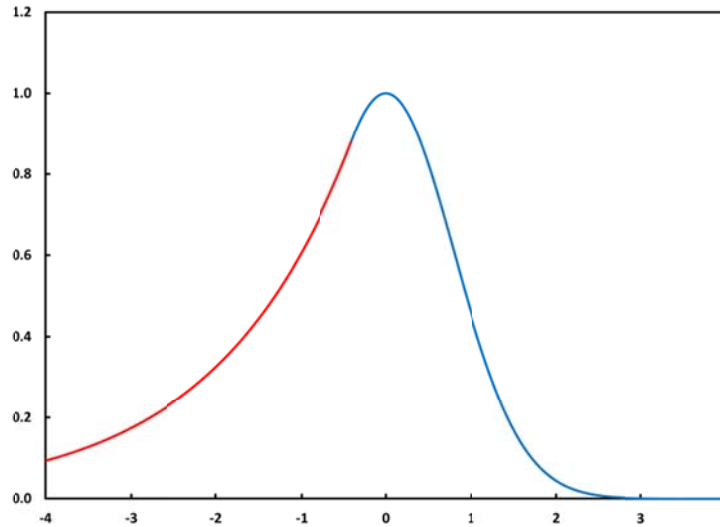
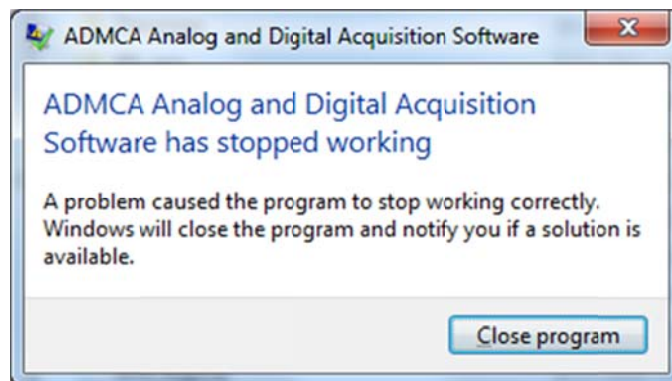


Figure 10. Analytic model for gamma-ray peaks.

Don't be too surprised if the following error message pops up on the screen while running the ADMCA program:



The data acquisition process is actually controlled by the Pocket MCA electronics module, not the desktop PC. Click the *Close program* button and restart the ADMCA icon whenever you need to get access to the data again. The Pocket MCA keeps logging the pulse height data to its internal memory even though the master program on the PC has shut down.

Experimental Procedure

There are two sets of measurements required for this experiment. The first is to measure the attenuation of the converter elements as a function of gamma-ray energy. The setup for this is shown in Figures 11a & b. The 10 μCi ^{226}Ra source is placed at the top of a 12" long acrylic tube. For the unattenuated measurement, insert the bottom of the tube in the white Delrin plate provided. For the measurements with absorbers in place, simply place the tube directly on top of each one. Please exercise some care to avoid dropping the source. As indicated, it's best to keep the source label side up.

The setup for measuring e^+e^- pair production is shown in Figures 12a & b. The critical trick of this procedure is to make sure that direct radiation from the Ra source never directly illuminates the Ge detector. The tungsten brick (with the white plastic handle) is extremely dense (**Don't Drop!!!**) and is quite effective as a shield. For this reason, keep the source at the back edge of the W block, as shown. The various converter plates should be placed on the inclined support directly above the Ge detector. Gamma-rays with energies above the $2m_e c^2$ threshold can convert to positrons and electrons and the positron will quickly slow down, capture an electron and annihilate into two 511 KeV γ -rays. This effective fluorescence is the signature that you will detect in the spectra taken with the six elemental converter plates. The conversion efficiency of this process is low so make sure you integrate for a long enough time to obtain sufficient statistics. **Also make sure to take a background run with no absorber!**

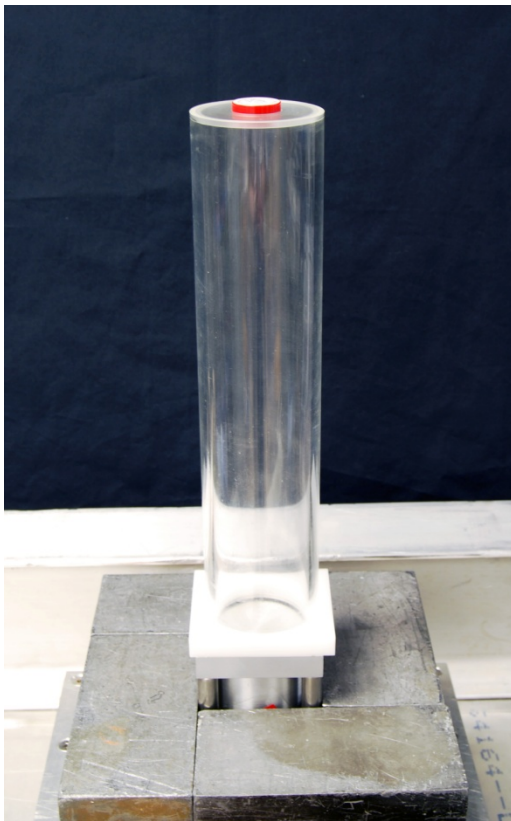


Fig. 11a. Unattenuated measurement of the ^{226}Ra spectrum.

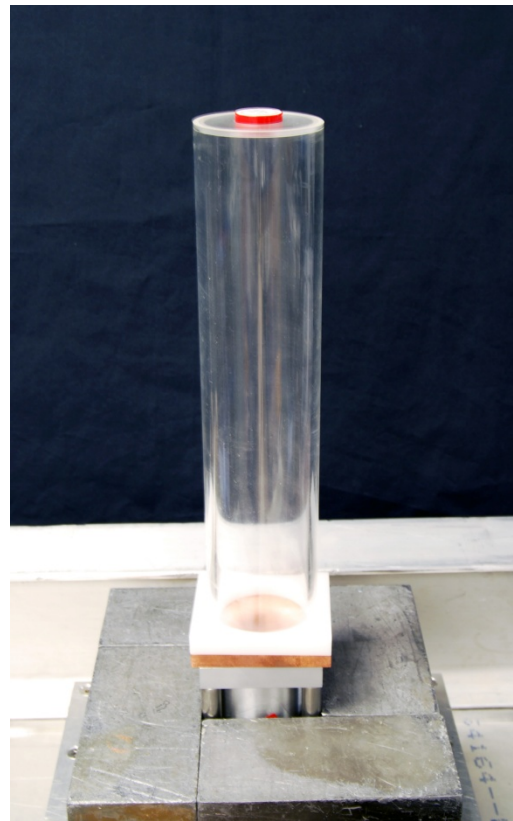


Fig. 11b Attenuated measurement of the ^{226}Ra spectrum with a Cu absorber.

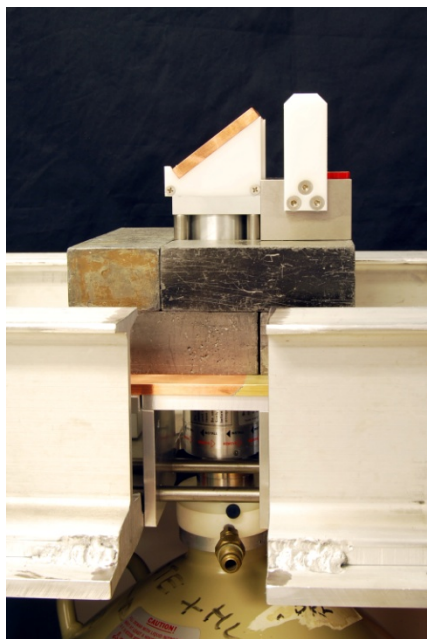


Fig 12a. Side view of the source, converter plate and detector housing.

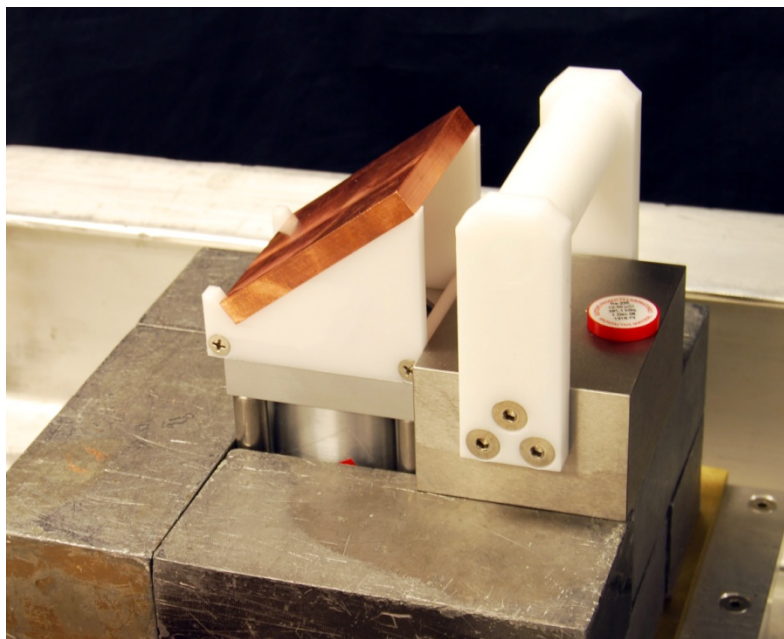


Fig 12b. Close-up view of the ^{226}Ra source and Cu converter plate.

Data Analysis

The first task for data analysis is to estimate the photon absorption cross sections for carbon and lead and compare these measurements with values obtained from the NIST Web site, <http://www.nist.gov/pml/data/xcom/index.cfm>. Accessing this URL will get you to the NIST XCOM: Photon Cross Sections Database. If you need instructions for use, go to option 4: *How to Run the XCOM Program*. For the less timid, proceed directly by clicking *Database Search Form* on the right-hand side of the screen, then click *Submit Information*. Make sure to select the “All quantities in barns/atom” option under *Options for output units*:. You will be interested in the spectral range from 290 KeV to 2500 KeV. Select the element of choice and click the second *Submit Information* button. From your data, you should be able to measure the count attenuation for at least 20 different spectral lines. To obtain the atomic cross sections, divide $\log(n_1/n_2)$ by the number of nuclei per unit area. Compare with the predictions obtained from the NIST facility. Comment on the cross section behavior in the energy range of this experiment and any qualitative differences between carbon and lead.

The central focus of this experiment is the measurement of the relative yield of positrons as a function of the converter element and its thickness. Determine the relative yield per number of nuclei per unit area and make a log-log plot of the results as a function of target atomic number, Z . Fit this data to the functional form shown below to find the best fit values for C and n . (Use *Solver* in Excel or an equivalent non-linear fitting algorithm.)

$$\sigma_{e^+e^-} = C \left[\log\left(\frac{183}{Z^{1/3}}\right) - \frac{2}{7} \right] Z^n$$

How does the value of n compare to the expectation based on quantum electrodynamics?

c	299792458 m/s
e	$1.602176487 \times 10^{-19}$ C
μ_0	$4\pi \times 10^{-7}$
ϵ_0	$(\mu_0 c^2)^{-1}$
m_e	$9.10938215 \times 10^{-31}$ kg = 510.999 keV
r_0	$e^2 / (4\pi\epsilon_0 m_e c^2)$
N_A	$6.02214179 \times 10^{23}$ mol ⁻¹

Table II. Useful constants and parameters.

Isotope	E_γ (KeV)	Rel Intensity
²²² Rn	186.211	
²¹⁴ Bi	53.228	1.2
²¹⁴ Bi	241.997	7.43
²¹⁴ Bi	258.87	0.524
²¹⁴ Bi	274.8	0.474
²¹⁴ Bi	295.224	19.3
²¹⁴ Bi	351.932	37.6
²¹⁴ Bi	487.09	0.422
²¹⁴ Bi	785.96	1.07
²¹⁴ Bi	839.04	0.587
²¹⁴ Po	609.312	46.1
²¹⁴ Po	665.453	1.46
²¹⁴ Po	703.11	0.472
²¹⁴ Po	768.356	4.94
²¹⁴ Po	806.174	1.22
²¹⁴ Po	934.061	3.03
²¹⁴ Po	1120.287	15.1
²¹⁴ Po	1155.19	1.63
²¹⁴ Po	1207.68	0.451
²¹⁴ Po	1238.11	5.79
²¹⁴ Po	1280.96	1.43
²¹⁴ Po	1377.669	4
²¹⁴ Po	1385.31	0.757
²¹⁴ Po	1401.5	1.27
²¹⁴ Po	1407.98	2.15
²¹⁴ Po	1509.228	2.11
²¹⁴ Po	1583.22	0.69

²¹⁴ Po	1661.28	1.15
²¹⁴ Po	1729.595	2.92
²¹⁴ Po	1764.494	15.4
²¹⁴ Po	1847.42	2.11
²¹⁴ Po	2118.55	1.14
²¹⁴ Po	2204.21	5.08
²¹⁴ Po	2293.4	0.305
²¹⁴ Po	2447.86	1.57
Pb K α	74.9694	
Pb K β	84.936	
m _e	510.9989	
²² Na	1274.53	
⁵⁴ Mn	834.848	
⁵⁷ Co	122.0607	
⁵⁷ Co	136.4736	
⁵⁷ Co	692.41	
⁶⁰ Co	1173.237	
⁶⁰ Co	1332.501	
¹⁰⁹ Cd	88.0336	
¹³³ Ba	53.1625	
¹³³ Ba	80.9971	
¹³³ Ba	160.6109	
¹³³ Ba	223.2373	
¹³³ Ba	276.3997	
¹³³ Ba	302.851	
¹³³ Ba	356.0134	
¹³³ Ba	383.848	
¹³⁷ Cs	661.657	
⁴⁰ K	1460.83	
¹³³ Ba Σ	437.0105	

Table III. Gamma-ray energies of various isotopes.
(This data is also available in a file,
pair_prod_lines.xlsx.)

Appendix A

The IDL *gamma_peaks* analysis code is stored in the *rtorres1* user account in the directory, *C:\Users\rtorres1\Documents\le+e- progs*. This keeps the code protected from random modifications. Two source code files are required, *gamma_peaks.pro* and *gamma_shape.pro*. The code can be compiled and saved to the executable file, *gamma_peaks.sav*, using the procedures outlined in *IDL_instr.pdf* and *IDL_instrux.txt*. Access to the SAV file must be permitted for student use, ie. *phys-advlab1* and *phys-advlab2*.

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