

# Measurement of the Muon Lifetime

Carl W. Akerlof  
September 10, 2009

## Overview

The only fundamental constituents of matter that are important for most of science are electrons, protons, and neutrons. It was thus a surprise when Carl D. Anderson and his student, Seth H. Neddermeyer, discovered<sup>1</sup> the muon in cosmic rays in 1936. This unexpected result was confirmed<sup>2</sup> independently by J.C. Street and E.C. Stevenson almost immediately. With the absence of strong interactions, the muon mass, roughly 200 times greater than the electron and 10 times smaller than the proton, make these particles extremely penetrating if sufficiently energetic. In this experiment, you will observe tens of thousands of cosmic ray-generated muons that slow down and stop in a large tank of liquid scintillator. With care, you will be able to determine the lifetime of a muon at rest to a precision of the order of one percent. In doing so, you will find evidence for muonic atoms formed by the capture of negative muons by carbon nuclei. Since muons in a vacuum can only decay into electrons, electron neutrinos and muon neutrinos, all fundamental particles, the mean lifetime is an important number that directly determines the weak force coupling constant,  $G_F$ , to high accuracy. This experiment stresses the importance of accurate time calibrations and requires a variety of statistical methods for extracting accurate results and determining their significance.

## Historical Background

One of the major successes of physics in the 20<sup>th</sup> century was the discovery and characterization of the fundamental constituents of matter. A number of important










1897 $e^-$ discovered	J. J. Thompson 
1911 Cloud chamber invented	C. T. R. Wilson 
1918 Proton discovered	Ernest Rutherford 
1932 Neutron discovered	James Chadwick 
1933 $e^+$ discovered	Carl D. Anderson 
1933 Fermi theory of $\beta$ decay	Enrico Fermi 
1937 $\mu$ discovered	Seth Neddermeyer, Carl D. Anderson
1956 $\nu_e$ discovered	Clyde Cowan, Frederick Reines, et al 
1962 $\nu_\mu \neq \nu_e$	L. Lederman, M. Schwartz & J. Steinberger 
1975 $\tau$ discovered	Martin Perl 
1991 only 3 lepton generations	four experiments at LEP and one at SLAC
2000 $\nu_\tau$ discovered	DONUT Collaboration (54 physicists)

Table I. Selective History of Elementary Particle Discoveries (  = Nobel prize )

milestones along this path are listed in Table I. Although we still don't know much about the nature of "dark matter", it seems that we have found all of the light Fermions known as leptons which includes the muon. These are characterized by three generations of doublets: electrons and electron neutrinos, muons and muon neutrinos and, finally, taus and tau neutrinos (plus their antiparticle images). The masses of the charged members are 0.510998910, 105.6583668 and 1776.84 MeV. The masses of the neutrinos are relatively much smaller but at least two of them are non-zero. The subject of neutrino mixing is an active area of elementary particle physics. From measurements of the decay width of the  $Z^0$  boson, there are no additional generations of leptons unless the associated neutrino has a mass greater than 45 GeV, half the mass of the  $Z^0$ .

Following the realization that an unobserved neutral particle was required to satisfy energy conservation in nuclear  $\beta$ -decay, Enrico Fermi developed a phenomenological model of weak interactions that describes a very large range of phenomena including muon decay. Fermi was also responsible for the name, "neutrino", after the original term, "neutron", was co-opted by James Chadwick for a quite different beast. For muons, the decay rate is related to the Fermi weak coupling constant,  $G_F$ , by the approximate equation:

$$\frac{\hbar}{\tau_\mu} = \frac{G_F^2}{(\hbar c)^6} \frac{(m_\mu c^2)^5}{192 \pi^3}$$

Because the mass is too low for more exotic channels, a  $\mu^-$  can decay only into an electron accompanied by an electron anti-neutrino and a muon neutrino. In the absence of strongly interacting particles such as neutrons or protons (or quarks), this relationship is the most accurate way of determining  $G_F$ . A more accurate equation includes the phase space effect of finite electron mass and radiative corrections<sup>4, 5</sup>:

$$\frac{\hbar}{\tau_\mu} = \frac{G_F^2}{(\hbar c)^6} \frac{(m_\mu c^2)^5}{192 \pi^3} f\left(\frac{m_e^2}{m_\mu^2}\right) \left(1 + \frac{3m_\mu^2}{5m_w^2}\right) \left[1 + \frac{\alpha(m_\mu)}{2\pi} \left(\frac{25}{4} - \pi^2\right)\right]$$

where

$$f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \log(x)$$

and

$$\alpha^{-1}(m_\mu) = \alpha^{-1} - \frac{2}{3\pi} \log(m_\mu / m_e) + \frac{1}{6\pi} \cong 136$$

Note that the decay rate is inversely proportional to the 5<sup>th</sup> power of the muon mass. Use this same formula to estimate the lifetime of the tau and compare with the measured value. Can you identify why the actual value is measurably shorter?

The discovery of the muon was something of a surprise, prompting the famous comment by I. I. Rabi, "Who ordered *that*?". It is reasonably fair to say that we don't have a much better answer 70 years later. An intriguing aspect of muons is that in the formation of muonic atoms, they significantly shield the nuclear charge, raising the possibility that muons could catalyze hydrogen fusion. This would have substantial

practical consequences if only the muons could live significantly longer. For another view of elementary particles, see Figure 1.

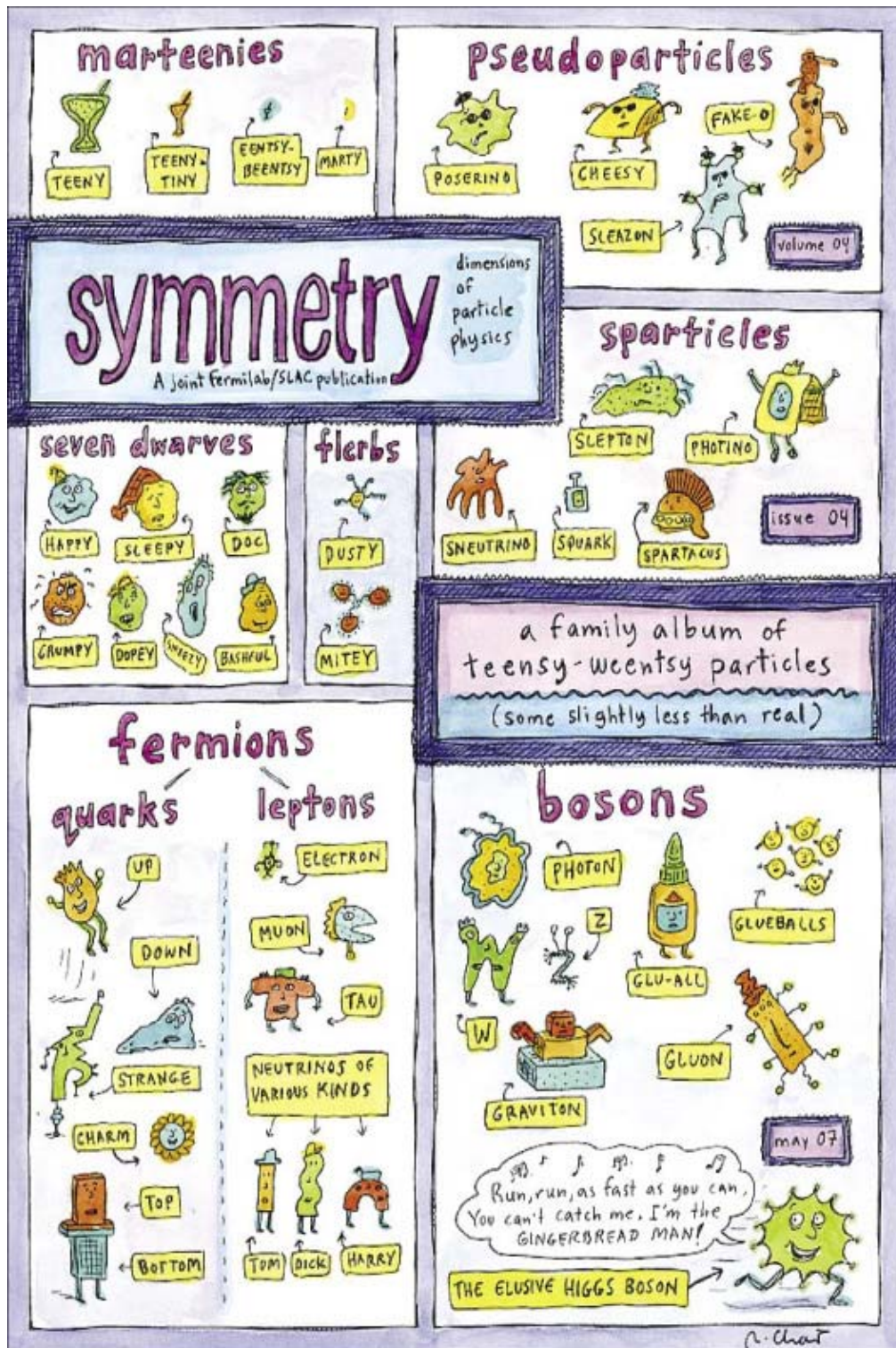


Figure 1. Elementary particle species according to Roz Chast<sup>6</sup>, *Symmetry*, April 2007

## Muons and Cosmic Rays

The source of muons in our laboratory is the interactions of protons with the nuclei of the atoms that constitute our atmosphere, principally nitrogen, oxygen and argon. The high energy cross-section for inelastic proton scattering on atomic nitrogen is  $2.65 \times 10^{-25} \text{ cm}^2$ . Averaging over the atomic constituents of air, the mean free path for inelastic interactions is  $89 \text{ g/cm}^2$ . We live at the bottom of this atmosphere at a pressure corresponding to  $1033 \text{ g/cm}^2$ . Thus, a primary proton must traverse 11.6 interaction lengths to arrive at the Earth's surface intact. That is good news – otherwise the radiation damage to life would preclude our own existence. For the muons that are observed in this experiment, the most likely height for production will be in the neighborhood of the first interaction length. The vertical distribution of the atmosphere is approximately exponential with a scale height,  $h_0$ , of 7600 m. Thus, one interaction length depth in the atmosphere corresponds to a height,  $h$ , determined by:  $89 = 1033 \cdot \exp(-h/h_0)$ . Solving, we find  $h$  must be 18,600 m above sea level. Travelling at the velocity of light, a particle will require  $62 \mu\text{s}$  to make the journey to Randall Lab. It will also be traversing  $1033 - 89 \text{ g/cm}^2$  of air. With an average energy loss of about  $1.8 \text{ MeV g/cm}^2$ , the muon must have an initial energy of about 1.7 GeV. Since the muon lifetime is  $2.197 \mu\text{s}$ , the  $62 \mu\text{s}$  travel duration is not so formidable since time dilation extends the apparent lifetime by the ratio of the muon energy to its rest mass of 0.105 GeV, a factor of 16.2 for this limiting case. Both the muon lifetime and the energy loss constraints point to initial muon energies of the order of 2 GeV or higher which require incident protons of 3 GeV or higher.

At these primary energies, most of the proton flux originates from outside the solar system. It is believed that the acceleration mechanism is predominantly shock fronts associated with supernova explosions in our galaxy. To get a sense of scale for this process, the interstellar gas number density is approximately  $1 \text{ cm}^{-3}$ . That translates to  $1.7 \times 10^{-24} \text{ g/cm}^3$  so the path length required to traverse a reasonable fraction of a nuclear interaction length is about  $10 \text{ g/cm}^2 \div 1.7 \times 10^{-24} \text{ g/cm}^3 = 6 \times 10^{21} \text{ m}$ . Travelling at close to the velocity of light, an energetic proton will wander for  $2 \times 10^{13} \text{ s}$  or 600,000 years before it has a significant collision. The diameter of our galaxy is about  $10^{21} \text{ m}$  so the free streaming residency time would be considerably less. However, the interstellar magnetic fields are more than sufficient to keep the protons confined to tortuous, highly folded paths within the Galaxy. Supernovae occur at a rate of about once per century so the cosmic ray flux at the top of our atmosphere represents the aggregate of about 10,000 such cataclysmic events.

A schematic diagram of a typical cosmic ray primary interaction is shown in Figure 2. An initial high energy proton or helium nucleus strikes a nitrogen or oxygen atom to produce a number of pions, both charged and neutral. The charged pions decay with proper lifetimes of 26 ns to muon and muon neutrino. The neutral pions only live for  $8 \times 10^{-17} \text{ s}$ , annihilating to two energetic photons. Except for ionization energy loss, the muons travel unimpeded through the atmosphere. For the gamma rays from  $\pi^0$ s, the evolution is more complicated. In approximately one radiation length,  $37 \text{ g/cm}^2$  in air, an energetic photon will interact with the nuclear electric field of nitrogen or oxygen to morph into two electrons that inherit the original energy. These daughter electrons and positrons will also interact with air on a distance scale of a radiation length to create bremsstrahlung photons, again dividing the energy. Thus after every radiation length

traversed, the original  $\pi^0$  is split into a factor of two more photons and electrons. As the average  $\gamma$ -ray energy drops below the pair creation threshold of 1 MeV, this process comes to a halt and normal ionization loss depletes the electron energies. Other particles besides pions are created in the primary cosmic ray interactions but they represent only a minor fraction of the total number or energy.

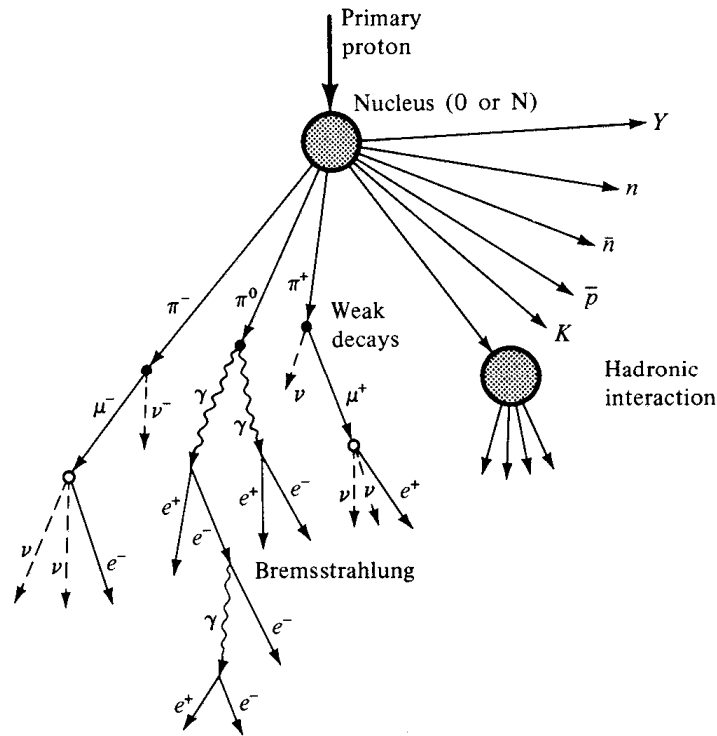


Figure 2. Schematic diagram of a cosmic ray interaction in the upper atmosphere (from Fraunfelder and Henley<sup>7</sup>).

The measured cosmic ray-induced muon flux is plotted in Figure 3. At the Earth's surface, the rate is of the order of several hundred per square meter per second. You will be able to confirm this in the present experiment. There is a slight preponderance of positive muons as would be expected from interactions of cosmic ray protons with the protons in atomic nuclei.

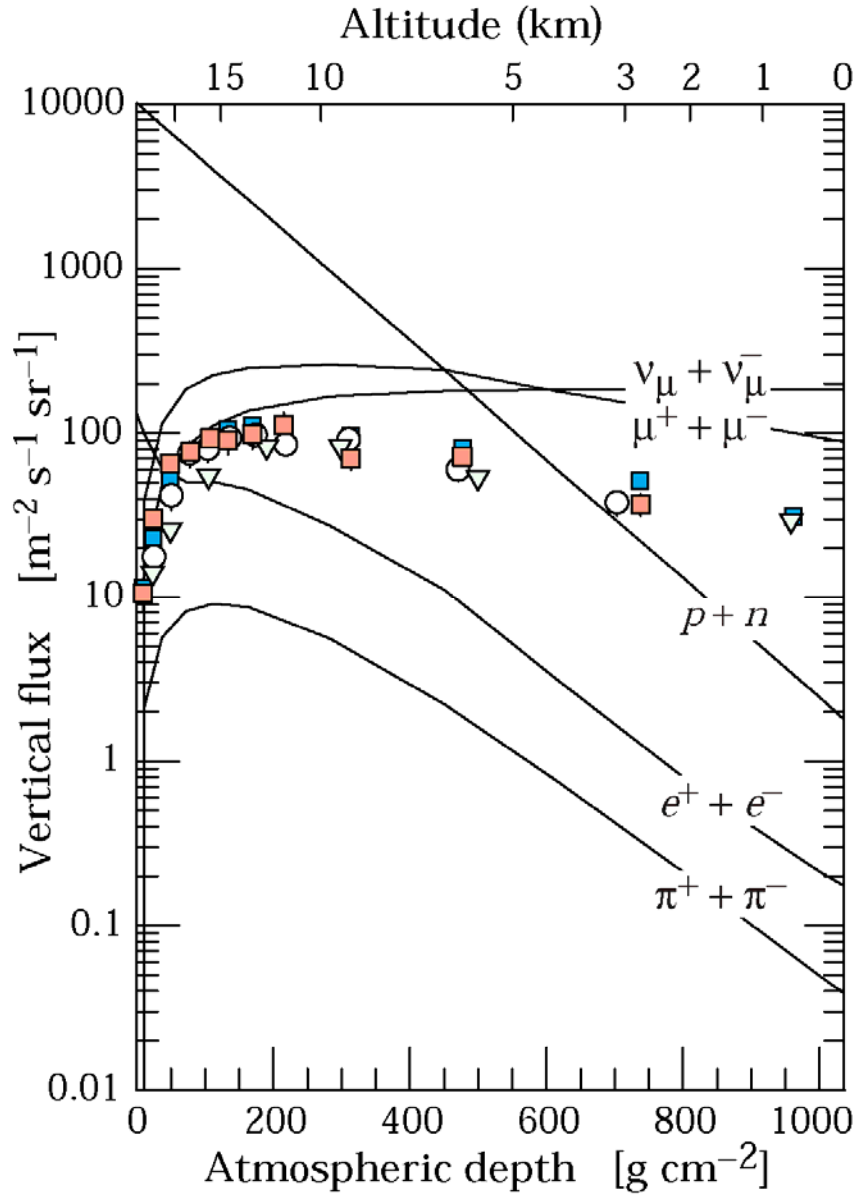


Figure 3. Vertical fluxes of cosmic rays in the atmosphere with  $E > 1$  GeV estimated from a power law approximation of the nucleon flux. The points show measurements<sup>8-11</sup> of negative muons with  $E_\mu > 1$  GeV (from the Particle Data Group<sup>12</sup>).

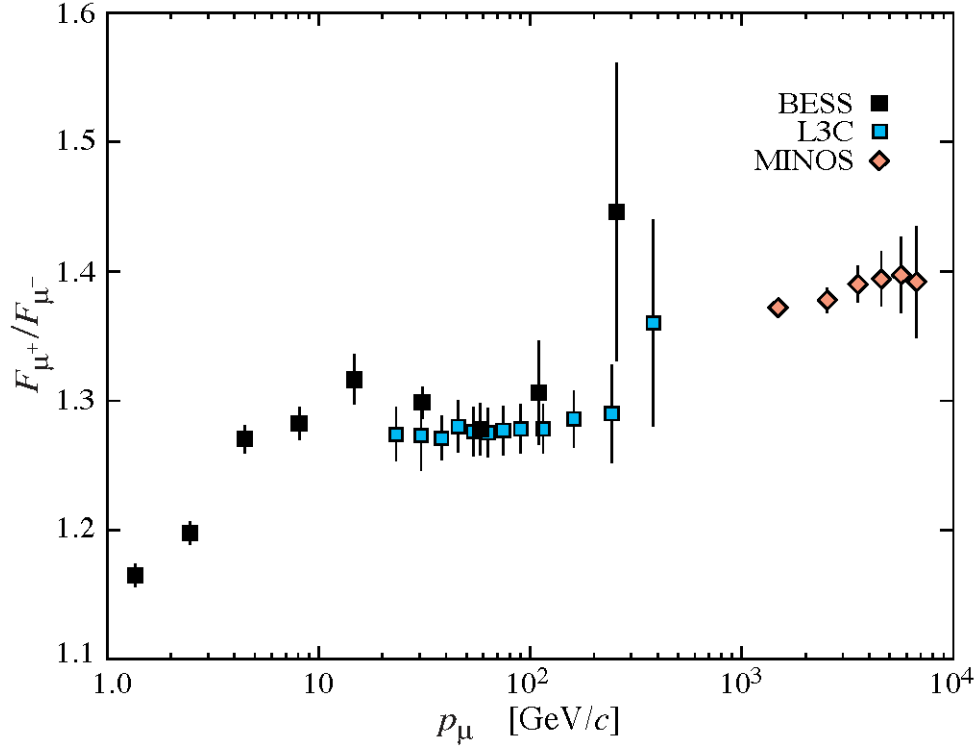


Figure 4. Muon charge ratio at sea level as a function of the muon momentum<sup>13-16</sup> (from the Particle Data Group<sup>12</sup>). The enhancement of positive muons at higher energies is due to the contribution of  $K^+$ .

### Equipment Description

The muon decay detection system was inherited from two very ambitious experiments that included substantial contributions from the University of Michigan. Both had origins in the notion of a Grand Unified Theory (GUT) of elementary particle physics and both required construction of large detectors deep underground for shielding against cosmic rays, *ie.* muons. The most promising idea was that quarks and leptons would be part of a larger multiplet, a representation of  $SU(5)$ , in analogy to the observed simpler structure of quarks or leptons alone. This suggested that quarks could mutate to leptons, just as muons decay to electrons and neutrinos. In particular, protons could decay to a positron and a neutral pion. Such a channel would be relatively easy to detect from the subsequent Čerenkov radiation in an optically transparent medium. There was great theoretical expectation that the proton lifetime would be no more than a few times  $10^{31}$  years. Consequently, a large cavern was excavated in the Morton salt mine near Cleveland, Ohio and filled with highly purified water surrounded by phototubes. The sensitive volume was  $20\text{ m} \times 20\text{ m} \times 20\text{ m}$ . (See Figure 5) Dan Sinclair, Jack van der Velde (both now retired) and Lawrence Sulak (now at Case-Western) were the three University of Michigan faculty members involved.



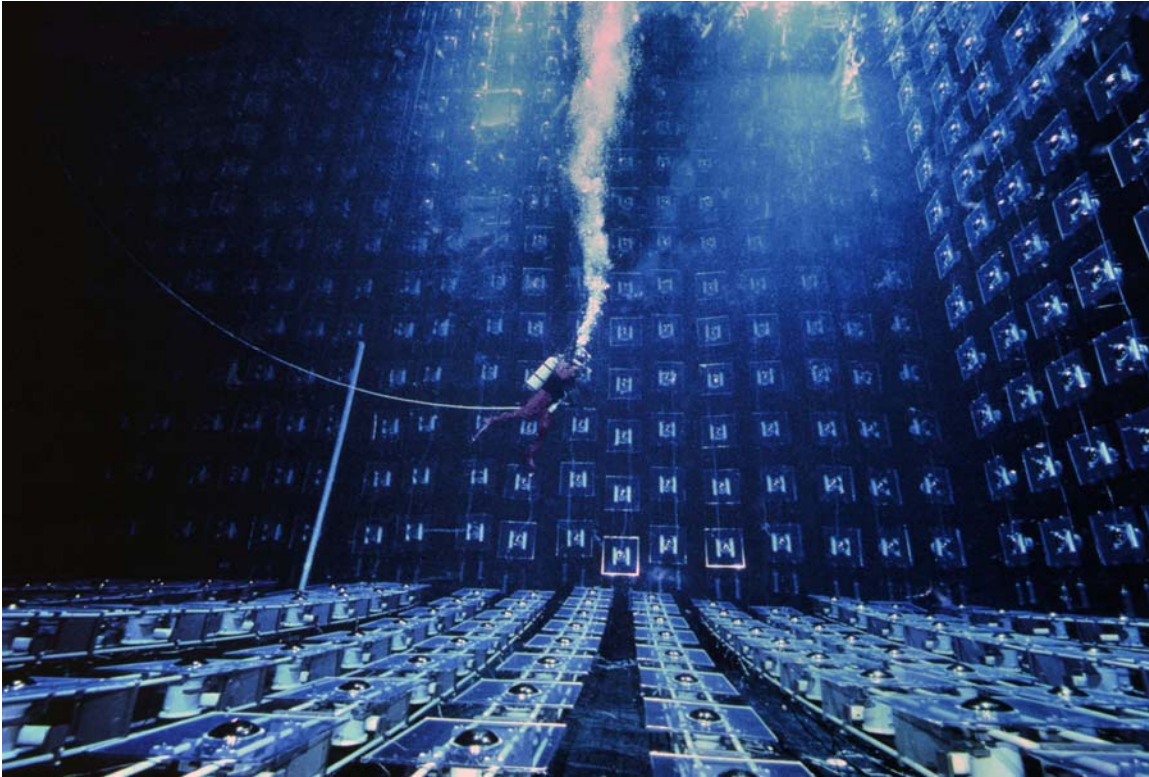


Figure 5 A view of the Irvine-Michigan-Brookhaven (IMB) detector. A diver is inspecting the phototube array. Square plastic plates with wavelength shifter surround the PMTs to increase the Čerenkov photon detection efficiency.

Unfortunately, Nature chose not to observe  $SU(5)$  symmetry so no protons decayed over a period of several years. As partial compensation, on February 23, 1987, the neutrinos from a blue giant supernova in the Large Magellanic Cloud swept past the Earth and 8 of them visibly interacted in the Irvine-Michigan-Brookhaven (IMB) detector<sup>17</sup>. (See the Powerpoint file by Jack van der Velde for a very nice description of this event: <http://www-personal.umich.edu/~jcv/SN1987aTalk.html>.) A few years later, the thin plastic liner that contained the purified water developed leaks and the whole experiment was decommissioned before the salt walls could collapse catastrophically. Two of the phototubes from the IMB were donated to the present muon lifetime experiment.

The mineral oil scintillator has been provided by the MACRO (Monopole, Astrophysics, Cosmic Ray Observatory) experiment whose local leader was Greg Tarlé. In this case, GUTs predicted the creation of primordial magnetic monopoles in the Big Bang that would continue to exist to the present day. The prime motivation for MACRO was the discovery of such objects<sup>18</sup>. The experiment required 600 tons of liquid scintillator<sup>19</sup> to look for electromagnetic interactions with low velocity, massive monopoles. No such particles were found and, worse yet, Nature did not provide compensation in the form of a nearby supernova event.

The general design of this experiment is quite similar to the description provided by Melissinos & Napolitano<sup>20</sup>. This text also contains a good explanation of the operation of photomultipliers, the active devices for particle detection. The liquid scintillator is a



tertiary system<sup>19, 21, 22</sup> dissolved in mineral oil. The primary scintillator is 1,2,4-trimethylbenzene (pseudocumene). The excitation of this molecule is transferred to 2,5-diphenyloxazole (PPO) which in turn radiates UV photons that are downshifted into the visible by *p*-bis(*o*-methylstyryl)benzene (bis-MSB).

The interior dimensions of the steel barrel containing the scintillator and phototubes are approximately 0.57 m diameter  $\times$  0.86 m high. As a rule of thumb, relativistic charged particles lose about 2 MeV per g/cm<sup>2</sup> as they traverse through matter. Thus, a muon that stops halfway through the scintillator will give up more than 86 MeV. The muon mass is 105.658 MeV and, on average, this energy will be distributed democratically among the three decay products,  $e^-$ ,  $\nu_e$  and  $\nu_\mu$ . Thus, there is considerable scintillation light from both the initial stopping muon and the subsequent decay. The Earth's atmosphere is about ten nuclear mean free paths in depth at sea level. From the approximation that the atmospheric pressure decreases exponentially with altitude with a characteristic scale height of 8 km, estimate the height above the Earth at which cosmic rays interact to produce the muons observed in this experiment. Given that the muon lifetime in its rest frame is 2.2  $\mu$ s, roughly how energetic must they be to survive the trip from near the top of the atmosphere to Randall Lab? Atmospheric pressure at sea level is  $1.01325 \times 10^5$  newtons/m<sup>2</sup> (Pascals) – how much energy do muons lose in their trip from near the top of the atmosphere to the Earth's surface due to ionization of air?

The two photomultipliers are fed high voltage from a single supply located on the left side of the NIM bin. To enable, turn on the NIM bin power on the right side, then the HV supply ON-OFF switch. The potentiometer should be permanently set at 0.4; turn the selector switch to 1 kV for a total of 1400 volts. **Power down in reverse order, making sure to ramp down the high voltage before switching off the unit.**

A critical aspect of this experiment is the accurate calibration of the delay time between muon entry into the scintillator tank and its subsequent decay. An Agilent (formerly Hewlett-Packard) 33120A Function/Arbitrary Waveform Generator has been provided for this purpose. After powering on, set the waveform to square wave (button 2) and adjust the amplitude to 2.0 volts after pressing button 7. Finally, adjust the square wave duty cycle to 20% by pressing the *Shift* button followed by the *% Duty* button (button 8). The output of the function generator must pass through an RC filter with a time constant of  $51 \Omega \times 680 \text{ pF} \approx 35 \text{ ns}$  before pulse shaping by a discriminator.

The signal processing for this experiment is fairly straightforward. Signals from both photomultipliers are converted to standard logic pulses by discriminators. These are fed to a coincidence circuit that determines when pulses from both tubes arrive simultaneously. The coincidence signals are sent to the *Start* and *Stop* inputs of a Time-to-Amplitude Converter (TAC) that in turn feeds a MultiChannel Analyzer (MCA). By delaying the arrival of the *Start* pulse by 67 ns, the logic assures that the TAC will be stopped by a second coincidence pulse occurring up to 10  $\mu$ s after the first, characteristic of muon decay. A dual channel scaler is wired to record the single and double coincidence events corresponding to the passage of a single muon and to the correlated decay.

## Experimental Procedure

The first step of this experiment is to accurately establish the relationship between time delay and the corresponding channel registered by the multichannel analyzer. Initialize the MCA software on the data acquisition computer and set it to acquire data. For the following procedure, also set the accumulation live time to 10 seconds. The signal processing logic should be wired as shown in Figures 6 and 7.

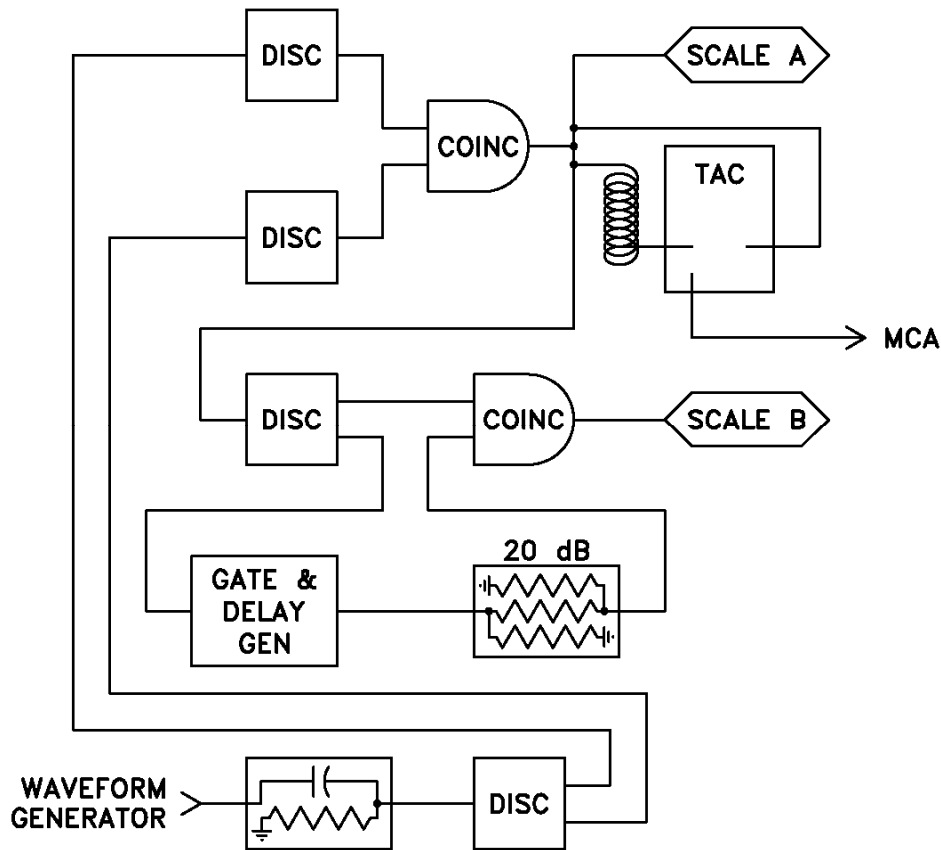


Figure 6. Schematic diagram for MCA calibration.

This setup is a good place to explore the behavior of the electronics. Use the Agilent 54642A 500MHz oscilloscope to display the signals generated by various stages of the signal processing chain, starting with the waveform generator, then discriminators, coincidence units and, finally, the TAC. Note the signal amplitudes and durations. Particularly for the TAC, examine the output amplitude as a function of the waveform generator frequency over the range from 0.1 to 10.0 MHz.

As you will soon learn, the response of the TAC departs significantly from linearity, particularly for short interpulse durations. This is also the most crucial segment of the data since the average muon lifetime is about  $2 \mu\text{s}$ . Thus, the optimum calibration sequence should not be uniformly distributed in interpulse time but instead reflect the enhanced importance of shorter intervals. A suggested sequence of intervals is:



three channels. With some imagination and mathematical prowess, interpolate each calibration peak to obtain a set of channel values corresponding to each delay time. Fit this data to the equation:

$$t = A_0 + A_{1/2} \cdot c^{1/2} + A_1 \cdot c$$

Plot the actual data and the fit curve obtained above. Examine the residuals to estimate the expected error of this interpolation formula over the range of the data.

The next major step is to test the performance of the entire system with random events. The TAC should be wired to start with the detection of a muon track and stop on a pulse from the waveform generator. As described earlier, the high voltage for the photomultipliers should be set at 1400 volts. The recommended waveform generator frequency is 20 kHz. If too low, the probability of getting pairs of pulses within 8  $\mu$ s is too small and if the frequency is much higher, the distribution of time intervals becomes distorted. The schematic and wiring diagrams diagram are shown in Figures 8 and 9. If all is operating correctly, you should obtain muon track detections at a rate of about 210 Hz (displayed in scaler channel A) and double events at about 34 Hz (displayed in scaler channel B or by the MCA software). Set the MCA live time duration to 50000 seconds or more and accumulate data overnight.

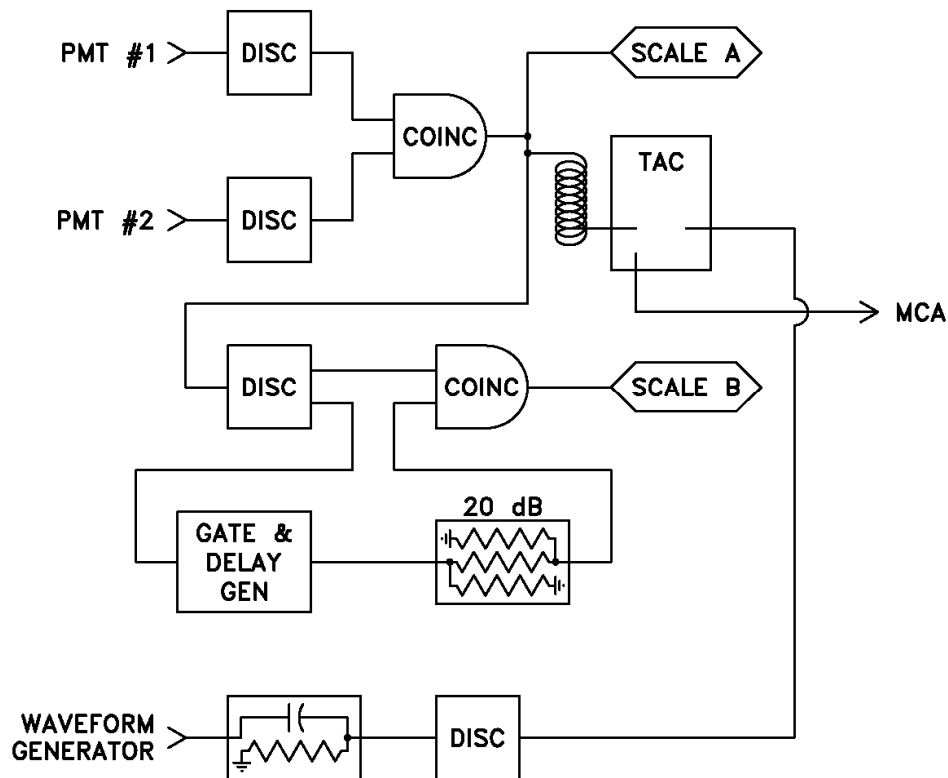


Figure 8. Schematic diagram for testing TAC & MCA performance.

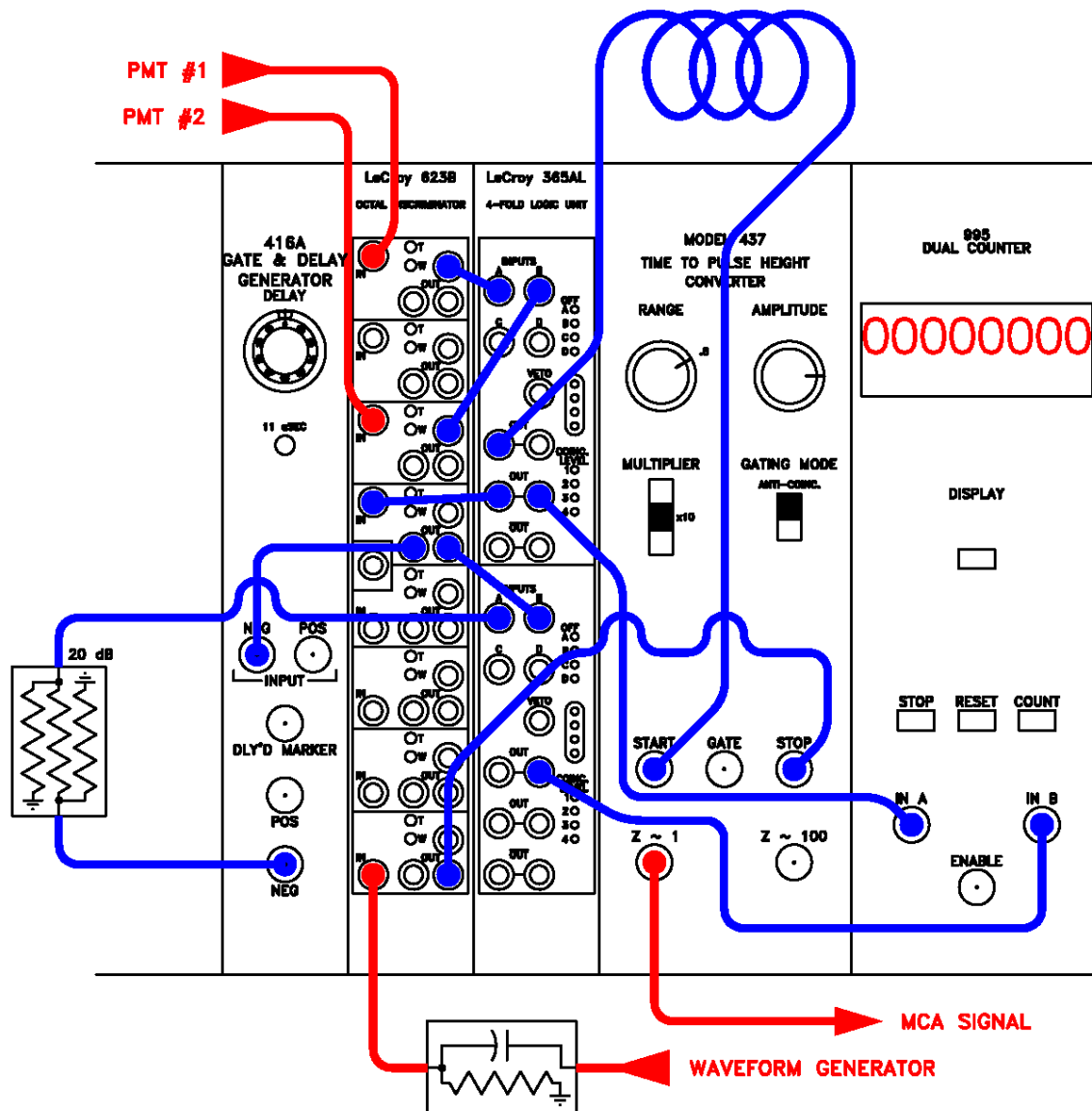


Figure 9. Detailed wiring diagram for testing TAC & MCA performance. Analog signals are **red**; digital signals are **blue**.

From the spectrum display provided by the MCA software, you will notice that the low end of the plot shows some rather ugly, erratic behavior. The high end also exhibits an abrupt cutoff somewhere below 2048, the maximum number of bins. Ignore these pathological extremes when analyzing the data. If you look more carefully at the data in the intermediate bins, you should also see a systematic decrease in amplitude, especially for the first few hundred channels. Copy the acquired data to Excel. Under the conditions suggested above, you will have accumulated around 1.7 million events distributed over 2000 bins for an average number per bin of 850 counts. Thus, the statistical accuracy for the amplitude of each bin is 3.4 % while the position of the events in each bin is known to 1 part in 2000 or 0.005 %. Such disparity of uncertainty is a poor way of rendering the information at hand. The solution is to rebin the data so that a



histogram of the data will have comparable accuracies along the horizontal and vertical axes. With a little thought, the optimum number of bins for a uniform distribution is  $N^{1/3}$  which in this case would be 119. The corresponding number of counts in each bin is  $N^{2/3}$ , approximately 14250, corresponding to a statistical accuracy of  $\sim 1/119$ . A generalization of this procedure should be applied whenever large, fine-grained data sets need to be graphically represented.

To test the response of the data acquisition system, first graph the binned data as a function of channel number. Include error bars so that the significance of any systematic trends can be assessed. Secondly, compute and plot the number of binned events per unit time using the channel to time calibration determined earlier. This should be a constant across the entire valid time range. In your plot, include error bars and a horizontal line showing the mean value. Compute the  $\chi^2$  statistic for this one-parameter representation of the data and find the statistical probability that the data is adequately described by this assumption.

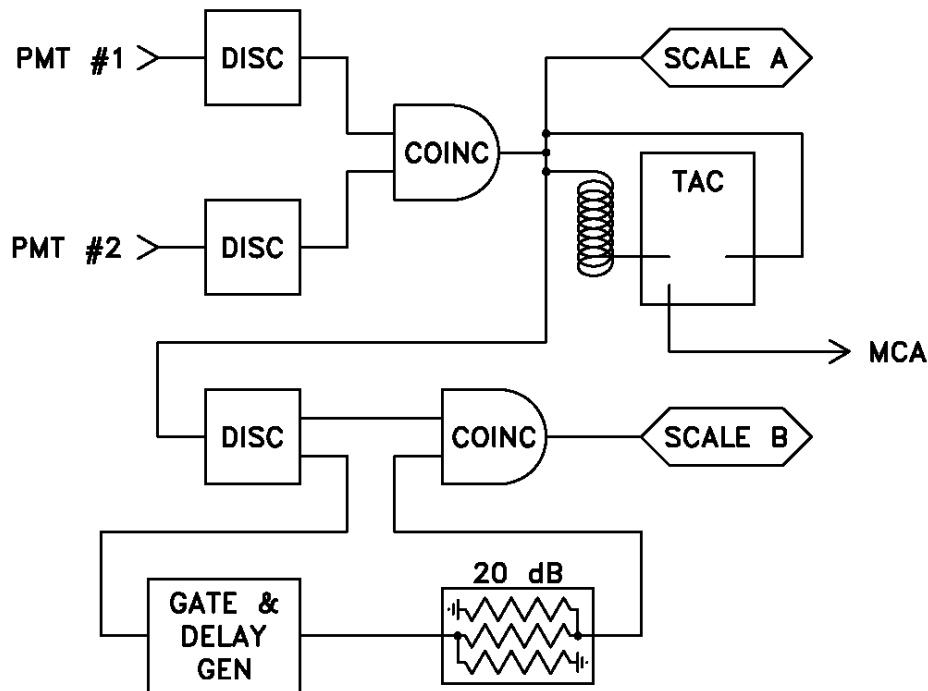


Figure 10. Schematic diagram for measuring muon lifetime.

You are now at the point to take actual muon decay data. Wire the electronics as shown in Figures 10 and 11 and power on the photomultipliers to 1400 volts. The lower coincidence unit is wired so that you can quickly observe the effects of the  $2.2 \mu\text{s}$  muon lifetime. By varying the coincidence delay, you can measure the effect on the coincidence rate monitored by channel B of the Ortec scaler unit. The width of the gate & delay generator pulse has been set at  $3.45 \mu\text{s}$  to optimize the ratio of muons to background. Vary the coincidence pulse delay time by adjusting the potentiometer labeled “DELAY” and plot the coincidence rate as a function of time. You will need an oscilloscope for calibration. Use integration times of at least 100 seconds to get convincing statistics.

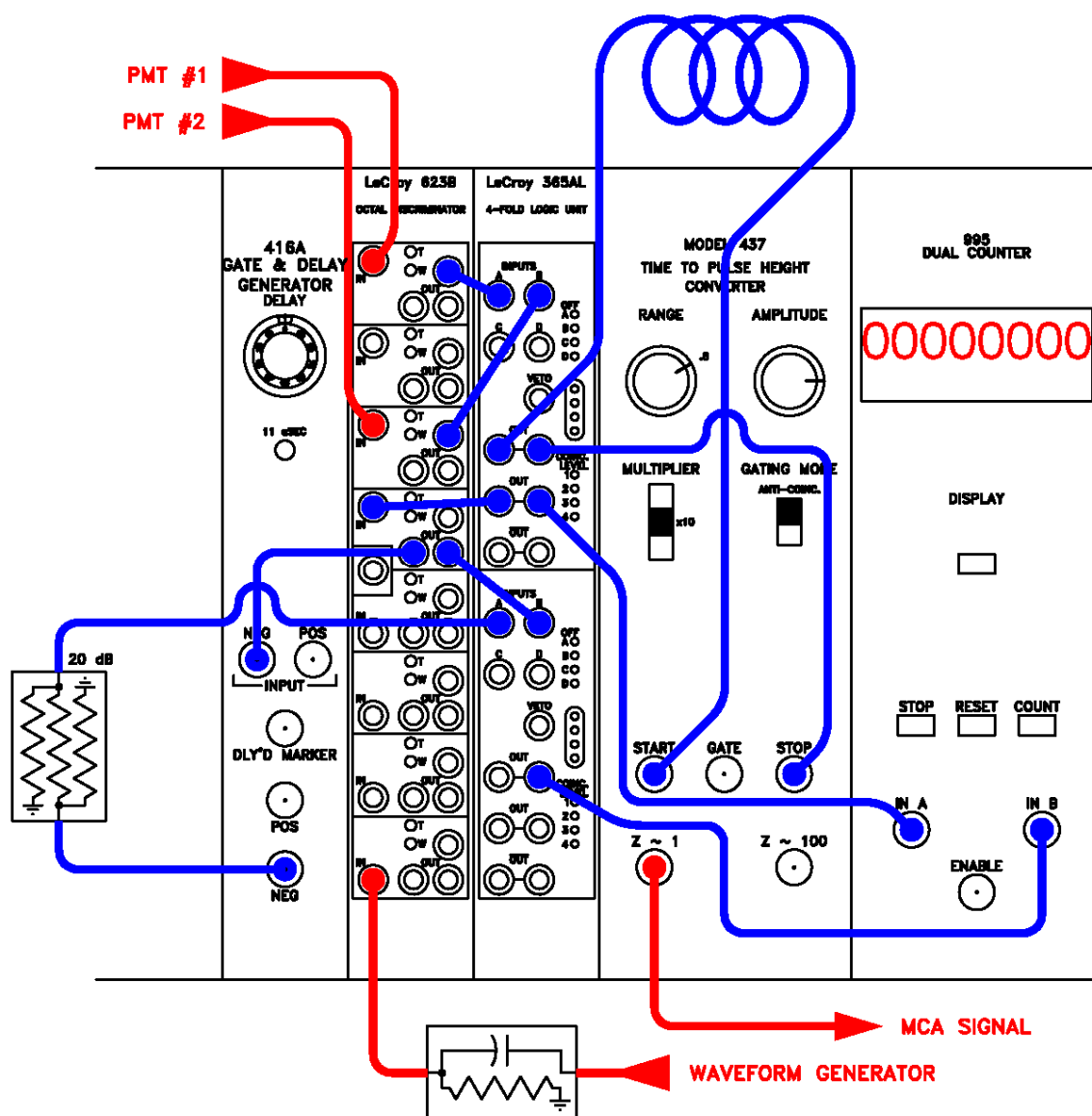


Figure 11. Detailed wiring diagram for measuring muon lifetime. Analog signals are **red**; digital signals are **blue**.

After ascertaining that the system is behaving properly, set the MCA integration time to something of the order of a day ( $= 86400$  s) and accumulate data. On completion, immediately copy and paste the digital data into Excel or any other application that can capture numeric text. After rebinning the data to match the statistical accuracies along the delay time and event count axes, fit the data to the form:

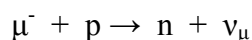
$$n(t) = A + B e^{-t/\tau}$$

Note that this requires a non-linear fitting procedure. Evaluate the  $\chi^2$  value and determine its significance. Use the method outlined in the Physics 441/442 statistics notes, Lecture

IV, to evaluate the error in the muon lifetime,  $\tau_\mu$ . Plot the data with error bars as well as the fitted curve obtained above.

The value just obtained for  $\tau_\mu$  is not quite the same as the muon vacuum lifetime. For negative muons, an additional decay channel is available that shortens the average life. This arises from the electrostatic attraction to a positive nucleus, sucking the muon into a hydrogenic quantum state. Because the muon is much more massive than an electron, the wavefunction is highly localized to the vicinity of the nucleus. For this experiment with mineral oil liquid scintillator, the predominant atom will be carbon. Estimate the characteristic muonic carbon 1s radius and compute the probability of finding the muon inside the nucleus.

For a negative muon in close proximity to carbon, the following reaction is possible:



This additional decay channel makes the lifetime for  $\mu^-$  about 7.8 % shorter in carbon<sup>23, 24</sup> than for  $\mu^+$ . For the cosmic ray muons of interest in this experiment, the charge ratio<sup>12</sup>,  $\mu^+/\mu^-$ , is approximately 1.15. These numbers are sufficient to compute a better estimate for the muon lifetime in vacuum. (Note that you should average the rate  $\propto 1/\tau$  rather than  $\tau$  itself. Imagine what happens if a particular value of  $\tau \rightarrow \infty$ .)

Compare your corrected result with the current best value posted by the *Particle Data Group* (<http://pdg.lbl.gov/>). Comment on the statistical significance of any difference and discuss possible important systematic errors in your measurements. Finally, compute the value of the Fermi constant,  $G_F/(\hbar c)^3$ , from your measurement of  $\tau_\mu$  and compare with the accepted value.

## References

1. Seth H. Neddermeyer & Carl D. Anderson, *Note on the Nature of Cosmic-Ray Particles*, Phys. Rev. **51**, 884-886 (1937).
2. J. C. Street & E. C. Stevenson, *New Evidence for the Existence of a Particle of Mass Intermediate Between the Proton and Electron*, Phys. Rev. **52**, 1003-1004 (1937).
3. E. Fermi, *Versuch einer Theorie der  $\beta$ -Strahlen. I*, Zeitschrift für Physik **88**, 161-177 (1934). An English translation is available: Fred L. Wilson, *Fermi's Theory of Beta Decay*, AJP **36**, 1050-1060 (1968).
4. Matts Roos & Alberto Sirlin, *Remarks on the Radiative Corrections of Order  $\alpha^2$  to Muon Decay and the Determination of  $G_\mu$* , Nuclear Physics **B29**, 296-304 (1971).
5. William J. Marciano, *Quantitative Tests of the Standard Model of Electroweak Interactions*, Annu. Rev. Nucl. Part. Sci.. **41**, 469-509 (1991).

6. Roz Chast, *Symmetry*, April 2007.
7. Hans Fraunfelder and Ernest M. Henley, *Subatomic Physics*, 2<sup>nd</sup> Edition, Benjamin Cummings (1991).
8. R. Bellotti *et al.*, Phys. Rev. **D53**, 35 (1996).
9. R. Bellotti *et al.*, Phys. Rev. **D60**, 052002 (1999).
10. M. Boezio *et al.*, Phys. Rev. **D62**, 032007 (2000).
11. S. Coutu *et al.*, Phys. Rev. **D62**, 032001 (2000).
12. Particle Data Group, *The Review of Particle Physics*,  
<http://pdg.lbl.gov/2009/reviews/rpp2009-rev-cosmic-rays.pdf>
13. BESS Collaboration (S. Haino *et al.*), Phys. Lett. **B594**, 35 (2004).
14. L3+C Collaboration (P. Archard *et al.*), Phys. Lett. **B598**, 15 (2004).
15. The MINOS Collaboration (P. Adamson *et al.*), Phys. Rev. **D76**, 052003 (2007).
16. M. Unger *et al.* (L3 Collab.), Inter. J. Mod. Phys. **A20**, 6928 (2005).
17. R. M. Bionta, *et al.*, *Observation of a Neutrino Burst in Coincidence with Supernova 1987A in the Large Magellanic Cloud*, Phys. Rev. Letters **58**, 1494-1496 (1987).
18. S. Ahlen, *et al.*, *Search for Slowly Moving Magnetic Monopoles with the MACRO Detector*, Phys. Rev. Letters **72**, 608-612 (1994).
19. M. Ambrosio, *et al.*, *The performance of MACRO liquid scintillator in the search for magnetic monopoles with  $10^{-3} < \beta < 1$* , Astroparticle Physics **6**, 113-128 (1997).
20. Adrian C. Melissinos & Jim Napolitano, *Experiments in Modern Physics*, 2<sup>nd</sup> Edition, Academic Press (2003).
21. Glenn F. Knoll, *Radiation Detection and Measurement*, 3<sup>rd</sup> Edition, John Wiley & Sons, Inc. (2000).
22. Konrad Kleinknecht, *Detectors for Particle Radiation*, 2<sup>nd</sup> Edition, Cambridge University Press (1998).

23. T. Suzuki, D. F. Measday, and J. P. Roalsvig, *Total nuclear capture rates for negative muons*, Phys. Rev. C **35**, 2212-2223 (1987).
24. M. Eckhause, T. A. Filippas, R. B. Sutton, and R. E. Welsh, *Measurements of Negative-Muon Lifetimes in Light Isotopes*, Phys. Rev. 132, 422-425 (1963).