# The Hall Effect and the Conductivity of Semiconductors

## 1.  Introduction

In 1879 Edwin Hall, a graduate student at Johns Hopkins University, observed that when a magnetic field is applied at right angles to the direction of current flowing in a conductor, an electric field is created in a direction perpendicular to both (see Figure 1).  The effect may be used to determine the sign of the moving charges that form the electric current.  The Hall effect is important in the analysis of semiconductor material (in which the charge carriers can be either positively or negatively charged).  It also has many important practical applications in detecting and measuring magnetic fields.  For example, electronic ignition systems for cars nowadays are based on Hall effect sensors, rather than mechanical breaker points used up to a few years ago.

One may consider the Hall effect to result from the force on a charge moving in a magnetic field.  Consider a positive charge moving in the direction indicated by the vector **v** in Figure 1.  With the direction of magnetic field **B** as shown, the force would be in the direction **vxB**.  Thus the far face of the conductor becomes *positively* charged.  On the other hand, if the current consisted of negative charges moving in the negative x-direction, the force would be in the direction **–(-vxB)**, which is the same direction as before, so now the negative charges would be pushed toward the far face of the conductor and it would become *negatively* charged.  The potential difference induced by this effect is called the Hall voltage, and the sign of the Hall voltage allows one to determine the sign of the carriers of current.

In studying the Hall effect it is useful to define various quantities.  The nomenclature follows that of Melissinos, pp. 83-88, except we use **B** for the magnetic fields, not **H**.

If the current is along the x-direction (Figure 1), the average velocity of the charge carriers is related to the electric field by
$$v_x = \mu E_x$$
The constant of proportionality $\mu$ is called the mobility, which is designated as $\mu_H$ when measured in a magnetic field.

The current density $J_x$ and the electric field are related as $J_x = \sigma E_x$ where $\sigma$ is called the conductivity. $J_x$ can also be related to n, the number of charge carriers per unit volume.  All the carriers in a box of length $v_x$ will pass through a unit area in 1 sec.  This corresponds to a volume $v_x$, and a total charge ne $v_x$.  Thus $j_x = nev_x$.  Substituting $\mu E_x$ for $v_x$ we get $J_x = ne\mu E_x$ and replacing $J_x$ by $\sigma E_x$ we get
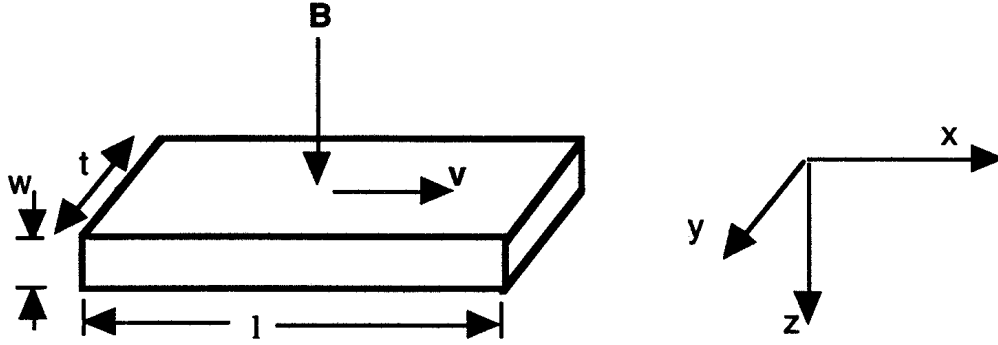
$$\sigma = ne\mu$$

Figure 1. The diagram shows the direction of the magnetic field **B**, and the direction of motion of the charge carriers that form the current.

The magnetic force on the moving charges is balanced by the electric field produced by the charges pressed against the near and far faces of the conductor $E_y=v_xB=\mu E_xB$. We define the Hall angle as

$$\phi = \frac{v_y}{v_x} = \frac{E_y}{E_x 1} = \mu Bt / 1$$

where 1 is the length and t is the thickness of the crystal.

Finally we define the Hall coefficient $R_H$ by the equation

$$E_y = |R_H| J_x B.$$

It is easy to show that $|R_H|\sigma = \mu$ (see Melissinos, p. 85); $R_H$ may be either positive or negative depending on the sign of the carriers of the current. It is an important property of the material carrying the current.

In this experiment we use the Hall effect to determine the sign of the charge carriers in samples of semiconductors and measure the electrical resistivity, the Hall coefficient, and the Hall mobility for each of two samples of germanium, one n-type, the other p-type. As discussed below, these quantities are strongly temperature dependent. The measurements are made over a range of temperature from approximately room temperature to 120°C. The samples are relatively pure by the standards of the semiconductor industry, through not as pure as the sample described in Melissinos. Nevertheless the samples should exhibit both *intrinsic* and *extrinsic* behavior, depending on the temperature. These terms are defined in the following paragraphs.


## A. Intrinsic Behavior

In a pure semiconducting substance such as silicon or germanium, there is an energy gap $E_g$ between the energy states of the bound electrons (called the *valence* band) and the states of free electrons (called the *conduction* band). This gap is ~1 eV, and in order to cross it an electron must receive this much energy by thermal excitation. The population density of electrons in the conduction band $n_c$ is therefore given by

$$n_c = n_v e^{-E_g/2kT} \tag{1}$$

where k is the Boltzmann constant and T is the temperature on the Kelvin scale. At room temperature kT~0.025 eV; thus only a tiny fraction, $\sim e^{-20} \approx 2 \times 10^{-9}$, are able to jump the gap. Though this fraction is small, it represents a fairly large density of electrons ($\sim 10^{12}$ cm$^{-3}$); this allows some moderate electrical conductivity, which is small in comparison to metals, but large in comparison to insulators. Hence the name *semiconductors*. One interesting property of pure semiconductors is that their electrical conductivity increases rapidly with increasing temperature according to Equation 1. With a very pure sample of semiconductor material (i.e., number density of impurities $\leq 10^{12}$ cm$^{-3}$) we would observe this *intrinsic* behavior <u>above</u> approximately room temperature.

## B. Extrinsic Behavior

The kind of semiconductors widely used in the electronics industry do not exhibit the intrinsic behavior described in the previous section. They have been deliberately "doped" with impurities to raise their conductivity. Figure 2a shows the energy level diagram for an n-type semiconductor. In
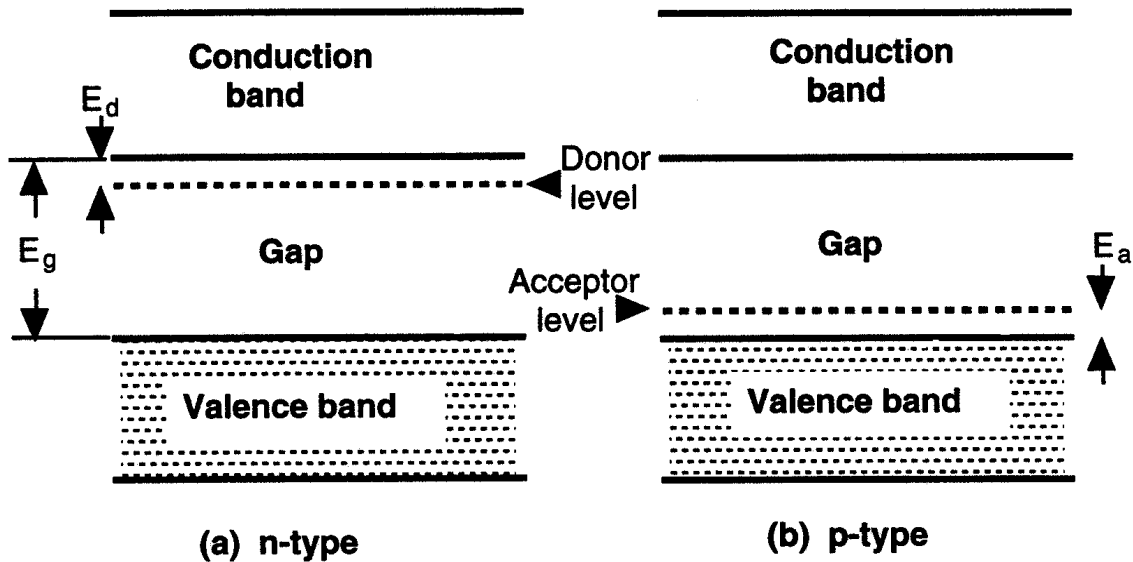


Figure 2. Energy band structure of *n*-type and *p*-type semiconductors.

case the impurities are *donor* atoms. They have an energy level which resides a fraction of an electron volt below the conduction band. At normal temperatures they readily cross this small gap $E_d$ to populate the conduction band. The density of electrons in the conduction band is limited only by the density of impurities in the sample. Typical samples might contain $10^{15}$ to $10^{16}$ atoms of dopant per cm$^3$. In a p-type semiconductor, the dopant is chosen so that the conduction band for the atoms of dopant lies a fraction of an electron volt above the top of the valence band as shown in Figure 2b. Electrons from the valence band readily cross the small gap $E_a$ to the acceptor sites provided by the dopant. In doing so they leave holes in the valence band. These holes behave like *positive* charges and account for most of the electrical conductivity in a p-type semiconductor, because of their higher mobility.

In the <u>extrinsic</u> region, conduction is due mainly to impurity carriers. The concentrations of impurities in samples of "pure" germanium are high enough that they will show extrinsic behavior at tem-

peratures below room temperature when the density of electrons in the conduction band is small compared to that of impurity atoms.

## 2. Equipment

The crystal, made by PHYWE, is mounted on a printed circuit card and has an internal electric heater monitored by a thermocouple. A constant current regulator mounted on the card provides constant control current, independent of the resistance of the crystal that varies strongly with temperature. This can be bypassed and the control current can be provided directly.

The 4 mm banana jacks bring out the Hall voltage and the drop of voltage along the crystal. The circuit card is mounted between the poles of an electromagnet to provide the magnetic field.
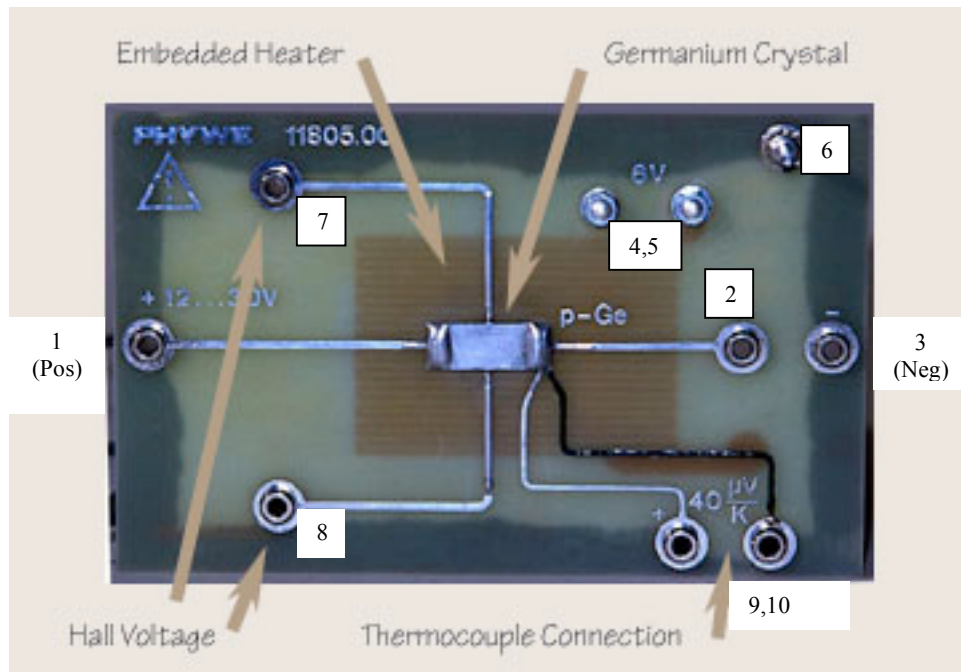


Figure 3. The PHYWE card with a germanium crystal. The Hall voltage is measured between terminals (7) and (8). The Hall current is applied between terminals (1) which is positive and (2) or (3); the latter uses the onboard current regulator. Heater current is applied between terminals (4,5), and the thermocouple connections are (9,10). The knob (6) controls the offset.

There is a thermocouple mounted on the board next to the crystal. The conversion factor to get temperatures from the voltage readings is 40 μV/°C. Note that the "reference junction" for this thermocouple is at <u>room temperature,</u> so you are measuring the temperature difference between the sample and room temperature which is about 20° C. **Therefore a sample temperature of 110° C corresponds to a thermocouple reading of about 3.6 millivolts**!

# 3. Objectives and Procedure

We want to measure the resistivity and the Hall coefficient of each sample as a function of temperature and to observe the Hall coefficient inversion of the p-type sample. Measurements will not be made below room temperature, so you will mostly see the <u>intrinsic</u> behavior. A heater is used to heat the samples. The magnetic field can be varied by varying the current in the electromagnet. The techniques and measurements are similar to those described in Melissinos I, Chapter 3, Section 3. One difference is that in our experiment we use a constant current source, and we determine the resistance of each element in the circuit by measuring the voltage drop across that element. A setting of ≈30 mA is appropriate for the current through the sample. Apply approx. 15 V between terminals 1 and 2 on the board. Make sure you observe the correct polarity; otherwise the current regulator on the board will not work. If the current is not ~30 mA, adjust the small "pot" with a screwdriver. The current through the sample should be monitored either with the current meter on the power supply or an external ammeter.

If the leads sampling the Hall voltage are not precisely opposite one another, the voltage they sample will not precisely change sign when the magnetic field is reversed because the voltage measured includes a component due to the IR drop between the two points. The adjusting knob (6 in Fig. 3) is used to adjust the Hall voltage to approx. 0 with no magnetic field. You can correct for any remaining offset by subtracting the Hall voltages measured without the magnetic field, or, better yet, measure the Hall voltages with both polarities of the magnetic field.

First, take data on the Hall voltage *vs.* magnetic field at room temperature with constant current through the sample. Center the sample between the pole tips of the magnet and orient it parallel to the pole tip. Measure the Hall voltage as a function of magnet current from zero to the maximum current (5 A). The magnet current can be read with sufficient accuracy from the meter on the supply. Use a Keithley microvoltmeter to read the Hall voltage. Also measure the voltage across the sample and the current through it, so you can calculate its resistance. Measure the Hall voltage for both directions of the magnetic field.

Now take data on the Hall voltage *vs.* temperature with constant current through the sample and the magnetic field at its maximum. For the thermocouple output, you can use a handheld DVM with a temperature scale to monitor the temperature, or use a millivoltmeter to measure the difference between the sample temperature and room temperature. *If you use the millivoltmeter, remember that the thermocouple reading should not exceed 3.6 millivolts.* Hook up a power supply capable of producing 5 or 6 V at about 1 A to the "6V" heater terminals on the board. Turn up the heater supply voltage to about 5 V and watch the sample temperature rise. When the temperature reaches 100° C, start decreasing the heater voltage. *Do not raise the temperature beyond 120°C.* Starting at about 110° C, gradually lower the sample temperature. While it is decreasing, monitor the Hall voltage with and without magnetic field and voltage across the samples and the temperature. You can also measure the Hall voltage *vs.* Hall current. To do this, use terminals (1) and (2) on the board to bypass the internal current regulator. Then use the power supply set up for constant current to vary the Hall current. Do not exceed 40 mA.

Take measurements for both n-type and p-type germanium. If this is a 3-week experiment, you should also do the copper sample. The procedure for copper is similar, but a much larger current is needed to see the effect. Our supplies can go up to about 7 A which should be sufficient. (See Question 4.) The temperature dependence for copper is not particularly interesting.

After completing all the measurements, calibrate the magnetic field at the center of the gap *vs.* current using a magnetic field probe (which probably uses the Hall effect!). Make sure you understand the direction of the magnetic field. You'll need the dimensions of the sample to calculate the resistivity.

In analyzing the data, generally follow the procedures in Melissinos I, pages 92-97. The Hall voltage is obtained directly from the difference between the 5 A and 0 A readings. Plot the corrected Hall voltages *vs.* magnetic field. Plot the resistance of the sample and Hall voltage *vs.* T.

a) From the slope of log $\rho$ *vs.* 1/T in the intrinsic region, estimate the value of the energy gap $E_g$ for germanium. ($\rho$ is the resistivity $\equiv 1/\sigma$; you can also plot log $V_{Sample}$ since it is proportional to $\rho$).
b) Verify which sample is n-type and which is p-type.
c) Determine the inversion temperature for the sample of p-type germanium.
d) From measurements of the Hall coefficients at the lower temperatures, estimate the density of impurity carriers in the sample (Melissinos I, p. 85). (This will not be very accurate since the data are not really in the extrinsic region).

### Additional Questions

1. Explain briefly the difference between intrinsic and extrinsic behavior. What determines which behavior applies at a given temperature?

2. Assuming each atom of copper contributes one conduction electron, calculate the density of charge carriers for copper and compare with that obtained for the semiconductor samples.

3. A sample of p-type germanium has a density of holes of about $10^{15}$ cm$^{-3}$. Assuming that the holes are the majority charge carrier in the material, what Hall voltage do you expect to measure when a sample of thickness 1 mm is placed in a 0.25 T magnetic field with a current of 10 mA flowing through it perpendicular to the magnetic field?

4. Estimate what Hall voltage you would expect for a copper sample with a current of 10 A and dimensions ~18 $\mu$m thick and 25 mm on a side. Why is the measurement of the Hall voltage more difficult with a good conductor like copper?

5. Explain how the Hall effect can be used to measure or sense magnetic fields.

# References

E.H. Hall, *American Journal of Mathematics,* vol. 2, **287** (1879).
H. Ibach, H. Luth, *Solid State Physics*, esp. Chap 12 & Panel XIV
C. Kittel, *Introduction to Solid State Physics*.
H. P. Myers, *Introductory Solid State Physics,* esp. Chapter 10.
A.C. Melissinos, *Experiments in Modern Physics*, esp. Chapter 3, Sec. 3
E.H. Putley, *The Hall Effect and Related Phenomena.*
S. M. Sze, *Physics of Semiconductor Devices*, Chapter 1.