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Measurement of Compton Scattering

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Whenever you look at yourself in a mirror or observe the refraction of light by glass or water, you are observing the interaction of electrons and electromagnetic waves. In these cases, the energy of individual photons is a few electron-volts while the rest mass of an electron is more than half a million eV. Recoil effects are negligible and the system can be described in terms of classical electrodynamics. For substantially more energetic photons such as those produced in radioactive nuclear decay, these conditions are no longer true and the physics is defined by relativistic kinematics and quantum electrodynamics. The interaction of such photons with electrons is called Compton scattering. In this experiment, you will verify the relativistic scattering kinematics to an accuracy of a few percent and compare the interaction cross section to the quantum electrodynamics (QED) prediction, the Klein-Nishina formula. By measuring the sum of the energies of the scattered photon and the electron recoil, you will also be able to observe the consequences of energy conservation.

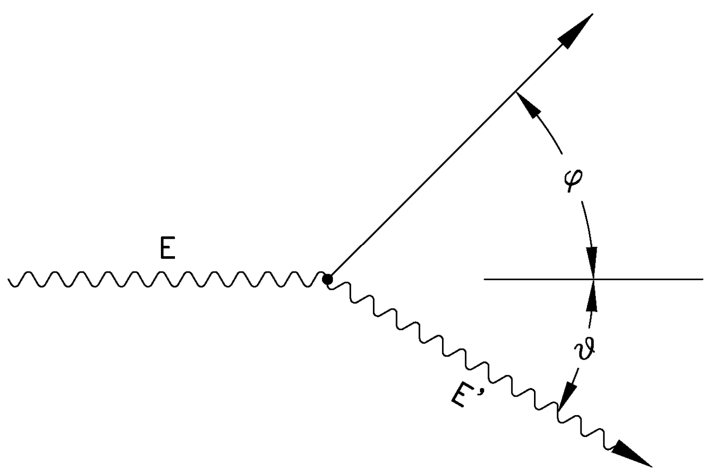

Figure 1. Kinematics of Compton scattering. An incident photon with energy, E, interacts with an electron at rest. Following the collision, the scattered photon has an energy, E', and is deflected by an angle, θ. The electron acquires a kinetic energy, E – E' and moves along a trajectory defined by the angle, φ.

By 1920, it was discovered that X-rays scattered from low-Z targets such as carbon exhibited a component that was clearly shifted to longer wavelengths. Initial attempts to explain this in terms of fluorescence of the scattering target could not account for actual observations. Arthur H. Compton realized that this could be understood as the result of relativistic scattering of the initial photon from a lightly bound atomic electron. For his insight, he shared the 1927 Nobel Prize in Physics with C. T. R. Wilson, inventor of the cloud chamber. Oskar Klein and Yoshio Nishina published the differential cross section for this process in 1927 using the techniques of relativistic quantum electrodynamics. (Klein was an assistant professor at the University of Michigan from 1923 to 1925 before returning to Europe.) The two lowest-order Feynman diagrams for this reaction are shown below in Figure 2.

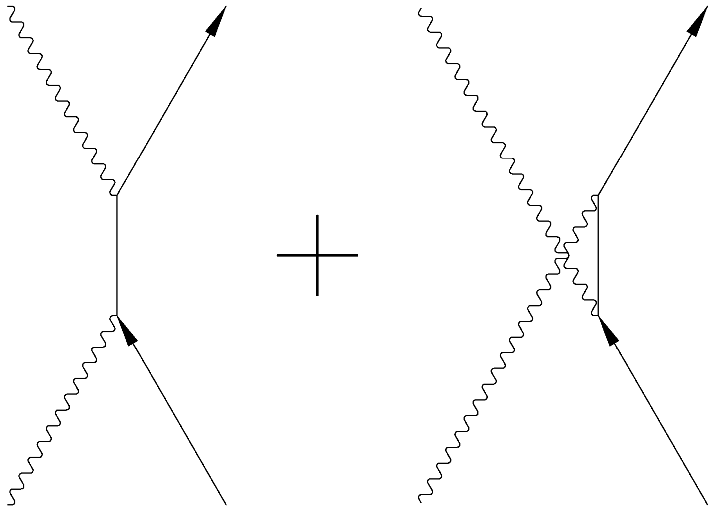

Figure 2. Lowest order Feynman diagrams for Compton scattering.

Compton Scattering Kinematics and Cross Sections

The kinematics of Compton scattering is easily derived from the conservation laws of energy and momentum under special relativity. The underlying assumption is that the target electron is free and at rest. This is a good approximation for atomic electrons in low-Z atoms for which the electron binding energy is substantially smaller than the incident γ-ray energy. It is convenient to express the kinematic quantities and cross sections in terms of two dimensionless ratios:

$$\varepsilon = \frac{E_\gamma}{m_e c^2}$$

where E_γ is the incident γ-ray energy and m_e is the electron rest mass and

$$f(\theta) = \frac{1}{1+\varepsilon(1-\cos(\theta))}$$

where θ is the scattering angle of the photon. The energy of the scattered photon is given by

$$E_\gamma^{'} = f(\theta)E_\gamma$$

and the kinetic energy of the scattered electron is:

$$T_e = E_\gamma - E_\gamma^{'} = \left(1-f\left(\theta\right)\right)E_\gamma$$

The differential cross section for this process is given by the Klein-Nishina formula

$$\frac{d\sigma}{d\Omega} = \frac{r_0^2}{2}\left\{ f(\theta) + f^{-1}(\theta) - \sin^2(\theta) \right\} f^2(\theta)$$

where r_0 is the classical electron radius, $\frac{e^2}{4\pi\varepsilon_0 m_e c^2}$. This expression can be integrated over 4π steradians to yield:

$$\sigma_{tot} = 2\pi r_0^2 \left\{ \frac{1+\varepsilon}{\varepsilon^2}\left[\frac{2+2\varepsilon}{1+2\varepsilon} - \frac{\log(1+2\varepsilon)}{\varepsilon} \right] + \frac{\log(1+2\varepsilon)}{2\varepsilon} - \frac{1+3\varepsilon}{(1+2\varepsilon)^2} \right\}$$

In the low energy limit, $\varepsilon \to 0$, the total cross section evolves to the Thomson limit:

$$\sigma_{tot} = \frac{8}{3}\pi\, r_0^2 .$$

c	299792458 m/s
e	$1.602176487\ \times 10^{-19}$ C
μ_0	$4\pi \times 10^{-7}$
ε_0	$(\mu_0 c^2)^{-1}$
m_e	$9.10938215 \times 10^{-31}$ kg $= 510.999$ keV
r_0	$e^2 / (4\pi\varepsilon_0 m_e c^2)$
N_A	$6.02214179 \times 10^{23}$ mol^{-1}
1 year	365.24219878 days
1 curie	3.7×10^{10} s^{-1}
E_γ (^{137}Cs)	661.657 keV
$\tau_{1/2}$ (^{137}Cs)	30.07 years
H atom. wt.	1.00794 amu
C atom. wt.	12.0107 amu

Table I. Useful constants and parameters.

Experiment Description

The equipment for this experiment consists of six major components: alignment plate, gamma-ray source, collimator, electron target and detector, gamma-ray detector, and signal processing electronics. Each of these is described in detail below.

Alignment plate

All of the components except the signal processing electronics are designed for mounting on the alignment plate, a ¾” thick PVC sheet. An aluminum disk with a fiducial notch has been machined to receive the Pb shielding container for the ^{137}Cs radioactive source. The gamma-ray collimator is bolted directly to the alignment plate and should be surrounded by Pb brick shielding blocks. Four vertical screws in a diamond array near the center of the plate are available for fastening the electron target/detector assembly. Finally 18 pairs of nylon pegs are spaced at 10º intervals from -10º to +160º to hold the gamma-ray counter assembly in accurate registration with respect to the gamma-ray source and electron target.

Gamma-ray source

The source of gamma-rays for this experiment is ^{137}Cs contained in a cylindrical Pb shield. The original activity was 1000 μCi quite a few years ago. Although now considerably less, it should still be treated with respect. We suggest that you ask one of the lab instructors to transport this item back and forth between the radioactive source safe and the experiment. Dropping the Cs source on the floor is considered a very bad idea. The arrow on the top of the source container indicates the position of the source material. Make sure that this arrow points away from the ¾” hole on the container side whenever it is handled. By the same token, the arrow should be turned to directly above the hole to enable measurements. ^{137}Cs is the ideal radioactive material for this experiment because the decay gamma-rays are monoenergetic and, at 661.657 KeV, are substantially greater than the electron rest mass energy while remaining below the pair production threshold of 2 m_ec^2. The ^{137}Cs decay is accompanied by a β^- with a maximum energy of 1176 KeV. To screen out this background, a 1/16” thick polyethylene absorber should be placed in the γ-ray beam at all times, either directly before or after the collimator.

Collimator

The direction of the incident gamma-ray beam is defined by a collimator consisting of a steel tube with 1” thick W apertures at either end. The collimator should be surrounded by 6 Pb shielding blocks to reduce stray radiation.

Electron target and detector

The electron target is a 0.787” thick plastic scintillator attached via acrylic light pipe to an 8575 photomultiplier tube assembly. The scintillator is basically polystyrene with added dopants that fluoresce in the presence of ionizing particles. The light pulses from a system like this are quite fast, of the order of a few nanoseconds. It is important that the target material contain only low Z elements such as H and C so that radiation scattering is dominantly from the Compton process.

Gamma-ray detector

The gamma-ray detector for this experiment is a 2" diameter NaI crystal attached to the end of a photomultiplier assembly (Canberra 802 2×2) with preamp (Canberra 2007P). The assembly is housed in a steel tube with a 1" diameter, 1" thick W aperture. A mounting spacer on the bottom of the steel tube mates with the nylon pegs on the PVC alignment plate. The light pulses from NaI have typical rise times of the order of 1 microsecond, 1000 times slower than the plastic scintillator described earlier. This has some annoying consequences when trying to cross-correlate the electron and gamma-ray signals. The high voltage should not exceed +1100 v.

Shielding

The counting rates in this experiment are not high so that the problem of stray radiation from the ^{137}Cs source or room background needs consideration. Lead bricks surrounding the steel collimator tube are essential. It is possible to reduce the background in the gamma-ray counter with lead bricks as well but at the cost of moving a lot of stuff (16 bricks) every time the detector is moved to a new scattering angle. Unless you find it essential, we recommend that you ignore extra shielding for the γ-counter at the expense of somewhat higher backgrounds underneath the scattering peaks. If you decide you need the shielding anyways, approach one of the instructors to get the appropriate instructions.

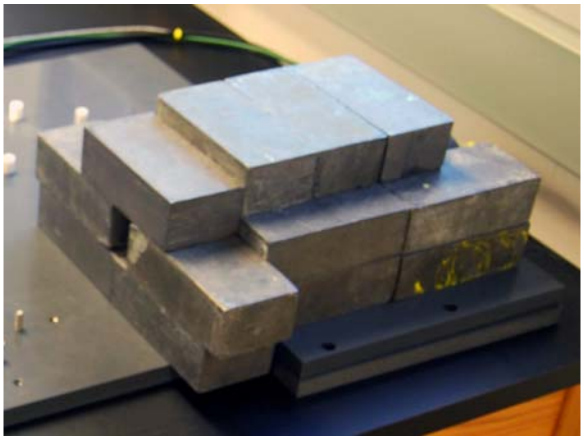

Figure 3. Lead brick shielding configuration for the γ-ray detector.

Signal processing electronics

The signal processing electronics are housed in a single rack-mounted NIM bin. The high voltage supplies for the photomultipliers are two double width modules on the far right-hand side. Both units obtain energy directly through external AC line cords rather than via the NIM bin so independent power switches can be found on the lower left corners. The voltages for both tubes must be positive. There is a fair amount of power available from these device – you definitely don't want to put your fingers near any open HV jacks. The electron detector PMT is powered by the left-hand unit. The high voltage for this unit should be set to +2600 volts. The electron detector PMT assembly has a significant gain shift with temperature. Thus, it's a good idea to turn on the HV power for at least an hour before using this component. The right-hand module supplies the gamma-ray counter – the voltage should be set to +960 volts.

855 Dual Spectroscopy Amplifier: The dual spectroscopy amplifier processes the signals from the gamma-ray and electron detectors. The upper channel is allocated for gamma-rays and the lower channel is used for electrons. This should not be swapped because the module is internally jumpered for positive and negative pulses respectively. The gamma-ray gains should be set to 2:50 Fine and 20 Coarse; the electron gains should be set at 6:00 Fine and 100 Coarse. Only the Unipolar outputs are used.

553 Timing Single Channel Analyzer: There are two single channel analyzer modules, one for the gamma-rays (on the left) and the other for electrons (on the right). In general, leave both *Window or Upper Level* knobs fully rotated clockwise. For most of this experiment, the *Lower Level* potentiometers can be set to the lowest values possible, fully counter-clockwise. The exception is for the total cross section measurements which require setting the gamma-ray threshold to just below the ^{137}Cs photopeak. The negative logic pulse from these units is used to trigger the gate & delay generator (see below).

416A Gate & Delay Generator: The gate & delay generator produces the appropriate gate pulses required by the Amptek multichannel analyzer. The amplitude is set by the inner knob and should be adjusted to produce a 3.5 to 4.0 volt positive pulse. Using an oscilloscope, the delay and width (outer ring) knobs should be set to window the analog pulses from the delay amplifier (see below).

427A Delay Amplifier: The delay amplifier is required to delay the detector analog pulses until the gating pulse decisions have been made by the single channel analyzers. For the situation where the recorded electron spectrum is conditional on a pulse in the gamma-ray counter, the required delay must be obtained by connecting two of these units in series.

871 Timer & Counter: The timer & counter module is useful for measuring the rates for the total cross section measurements. The time interval is set by the *Inc M* and *Inc N* buttons which sequentially advance two scale-of-ten counters. The time interval is $M\times10^N$ in units of 0.1 sec, assuming that the *Time Base Select* is set appropriately. Turn the *Dwell* knob completely counter-clockwise to prevent automatic recycling.

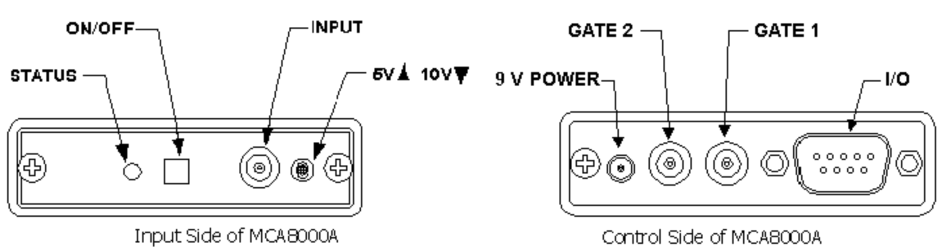

Figure 4. The front and back panels for the Amptek MCA8000A multichannel analyzer. This drawing is taken from the Amptek Web pages, http://www.amptek.com/mca8000a.html

Amptek MCA8000A Multichannel Analyzer: The multichannel analyzer accepts analog signals and converts these into digital counts for transfer to a computer. A gate input may be used to control which signals are processed. A front panel switch should be kept in the *down* position to maintain a 0 – 10 volt dynamic signal range. The gate pulse should be plugged into the *GATE 2* receptacle, adjacent to the 9 v power line.

Experimental Procedure

Before attempting any experiment, it is a good idea to calculate what to expect. Without some prior idea of the magnitude of the likely signal, you can waste a good deal of time.

1. The ^{137}Cs source was estimated to have an activity of 1000 μCi on January 11, 1965. Compute the number of disintegrations per second at the current date using the known half-life.

2. The initial γ-ray beam is collimated by a 1.070” diameter tungsten aperture at a distance of 7.291” from the source. Based on the decay rate computed above, what is the flux of ^{137}Cs γ-rays contained in this beam?

3. Calculate the scattered γ-ray photon energy for every 10º from 0º to180º.

4. Compute the Klein-Nishina differential cross section for every 10º from 0º to 180º using the formula given above. Also compute the integrated total cross section over 4π steradians.

5. The γ-ray target consists of polystyrene scintillator with an effective path length of 1.113” when angled at 45º or 135º with respect to the incident beam. Assuming the material has a density of approximately 1.05 gm/cm^3 and has a composition of $(C_8H_9)_n$, compute the fraction of γ-rays that will interact via Compton scattering.

6. The solid angle for detecting scattered γ-rays is defined by a 1” diameter circular tungsten aperture at a distance of 10.0” from the polystyrene target. Compute the expected flux for every 10º from the information computed in the steps above.

7. The energy of the scattered electron, T_e, is exactly equal to the energy loss of the initial photon. Thus, $dT_e = -dE'_\gamma$. From this consideration, calculate the differential cross section for the electron with respect to energy, $d\sigma/dT_e$, using the chain rule, $(d\sigma/d\Omega)\cdot(d\Omega/dE'_\gamma)$ and the expression for the solid angle, $\Omega = 2\pi\,(1 - \cos(\theta))$. Graph the result – you will want to compare this with actual measurements.

Measurements

1. Explore how the electronic works.

2. Measure the Compton scattering total cross section by measuring the γ-ray beam attenuation as a function of the thickness of zero to four ¼” polyethylene targets, $(CH_2)_n$,

placed just downstream of the beam collimation tube. Remove the electron detector assembly before attempting this measurement. Define the γ-ray signals using the potentiometer on the SCA, etc. Perform identical measurements with graphite (pure carbon). Test whether the interaction probability scales with target electron density or target nucleon density (ie. mass). To complete these calculations, you will need to measure the thicknesses of the scattering targets as well as their masses. It will be important for later measurements to know the calibration of the multichannel analyzer channels with respect to energy so make sure to accurately determine the location of the observed photopeak from ^{137}Cs.

3. Measure the differential energy spectrum for electrons. To remove extraneous backgrounds, make two separate measurements, first with the target counter angled at 45º to the incident γ-ray beam (as shown in Figure 5a) and, second, with the counter normal to the beam (as shown in Figure 5c). By subtracting the second spectrum from the first, you should obtain a plot that looks very much like the curve computed earlier from geometry and the Klein-Nishina equation. To compare these results on a uniform energy scale, obtain an energy calibration for the electron detector from section 5 below. The MCA need not be gated for these measurements. Make sure that the electron detector has been powered for at least an hour prior to accumulating data. Compare the theoretical Klein-Nishina values for $d\sigma/dT_e$ (see item 7 on previous page) with your measured distribution of dN/dT_e ..

4. Measure the scattered gamma-ray energy as a function of gamma-ray scattering angle. For small angles, take the difference between target in and target out. At larger angles, use an MCA gate pulse derived from the electron counter. Plot the peak of the scattered photon energy spectrum as a function of angle and compare with the curve expected from relativistic kinematics. Also compare the observed count rates at each angle with the expectations based on the calculations in items 1-6 on the previous page.

5. Measure the scattered electron energy as a function of gamma-ray scattering angle. Compute and graph the sum of scattered gamma-ray and electron energies as a function of the γ scattering angle. To what extent is energy conservation confirmed? To determine the energy scale for the electron detector, assign the position of the electron peak associated with the largest observed γ scattering angle to the expected electron recoil energy and assume a linear correspondence for all lower energies.

6. Earlier, it was stated that the scattered gamma-ray energy spectrum is determined solely by the dimensionless parameter, ε, the ratio of incident photon energy to electron rest mass. Suppose that this value is allowed to float. Assuming that the peak spectrum channel for each scattering angle is proportional to the scattered energy, one can find the value of ε that makes this relationship as close to linear as possible, assuming an intercept at the origin. Compare this estimated value of ε with the known ratio for the ^{137}Cs γ-ray.

7.

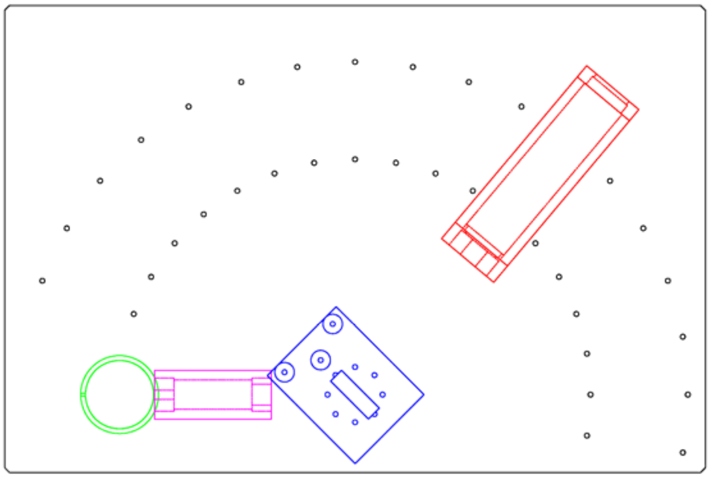

Figure 5a. Orientation of electron counter for γ-ray scattering angles between 0º and 90º.

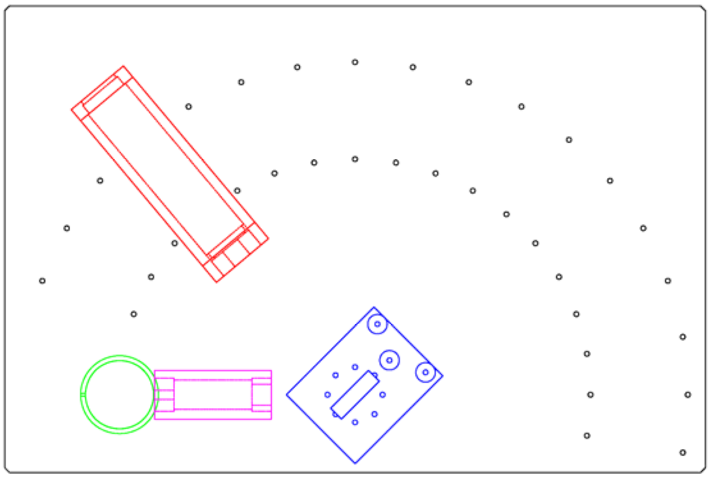

Figure 5b. Orientation of electron counter for γ-ray scattering angles 90º or greater.

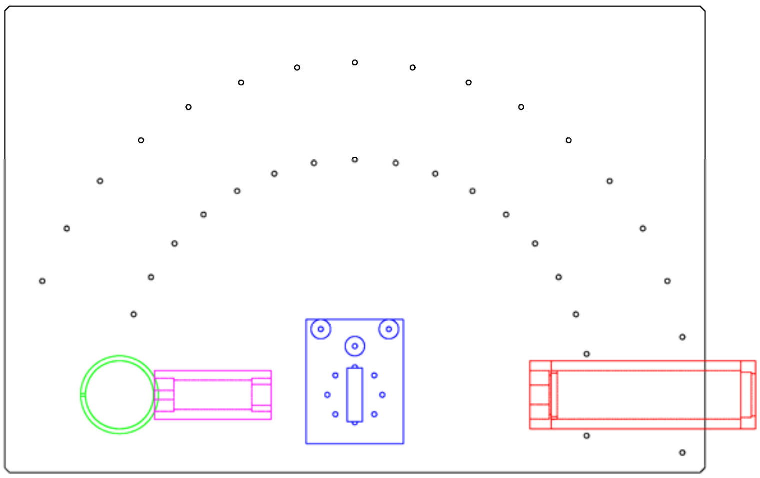

Figure 5c. Orientation of electron counter for measuring the Klein-Nishina differential cross section. The electron counter is first mounted as shown in 5a and then rotated by 45º as shown here to perform a difference measurement. The gamma-ray counter is not used for this part of the experiment.

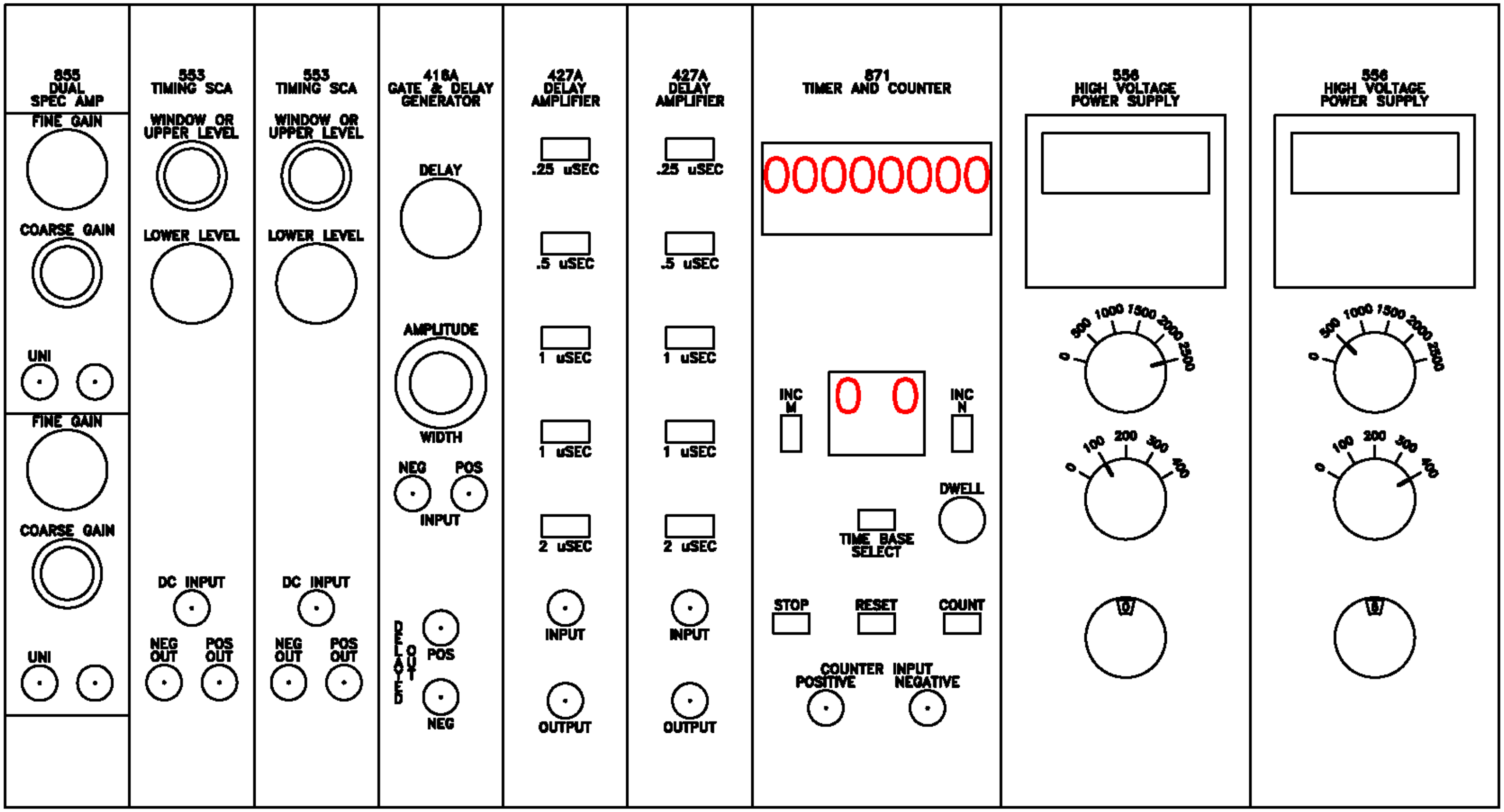

Figure 6. NIM signal processing electronics modules

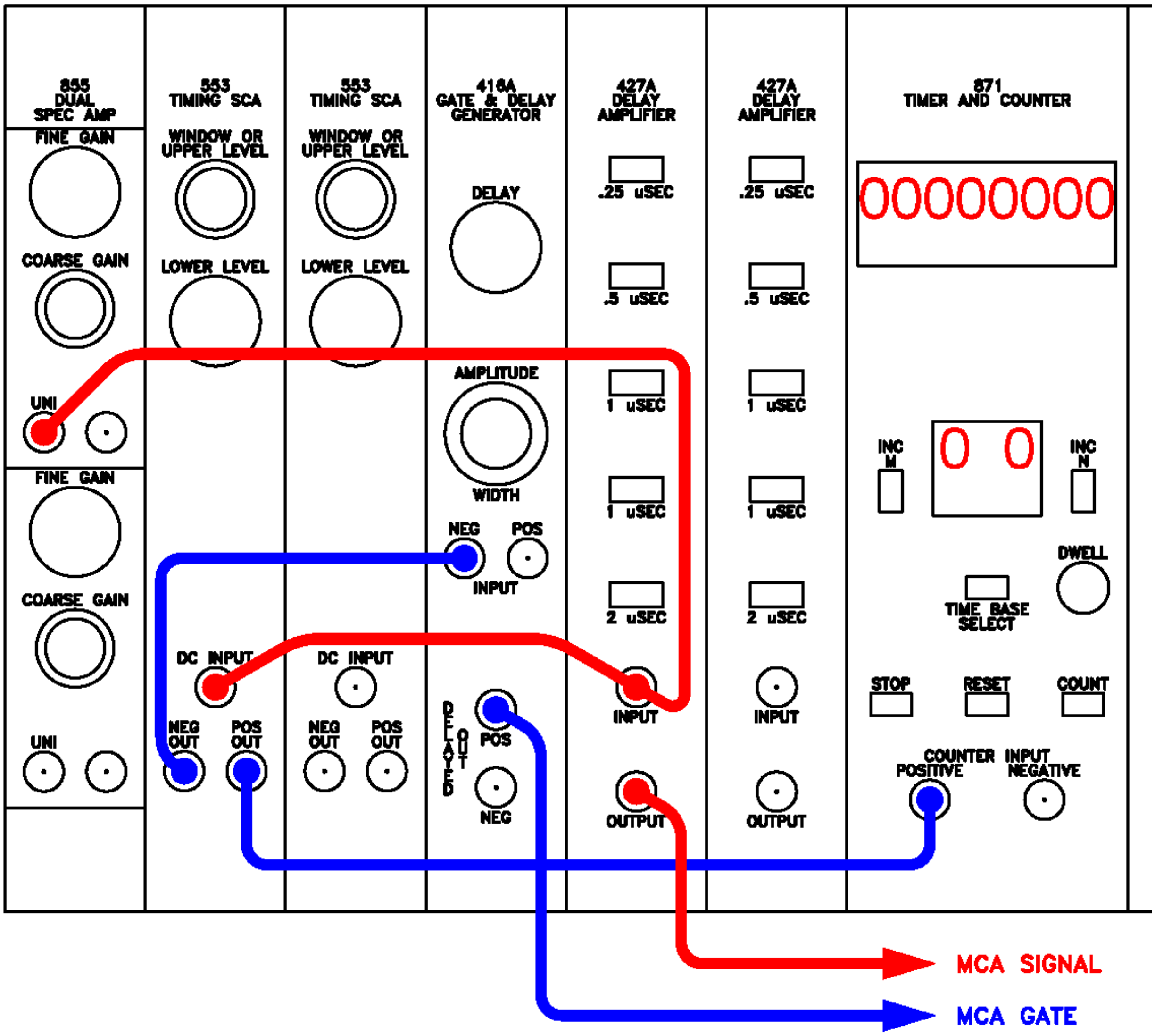

Figure 7. Wiring for Klein-Nishina total cross section measurements. **Red** cables carry analog signals; **Blue** cables carry logic levels.

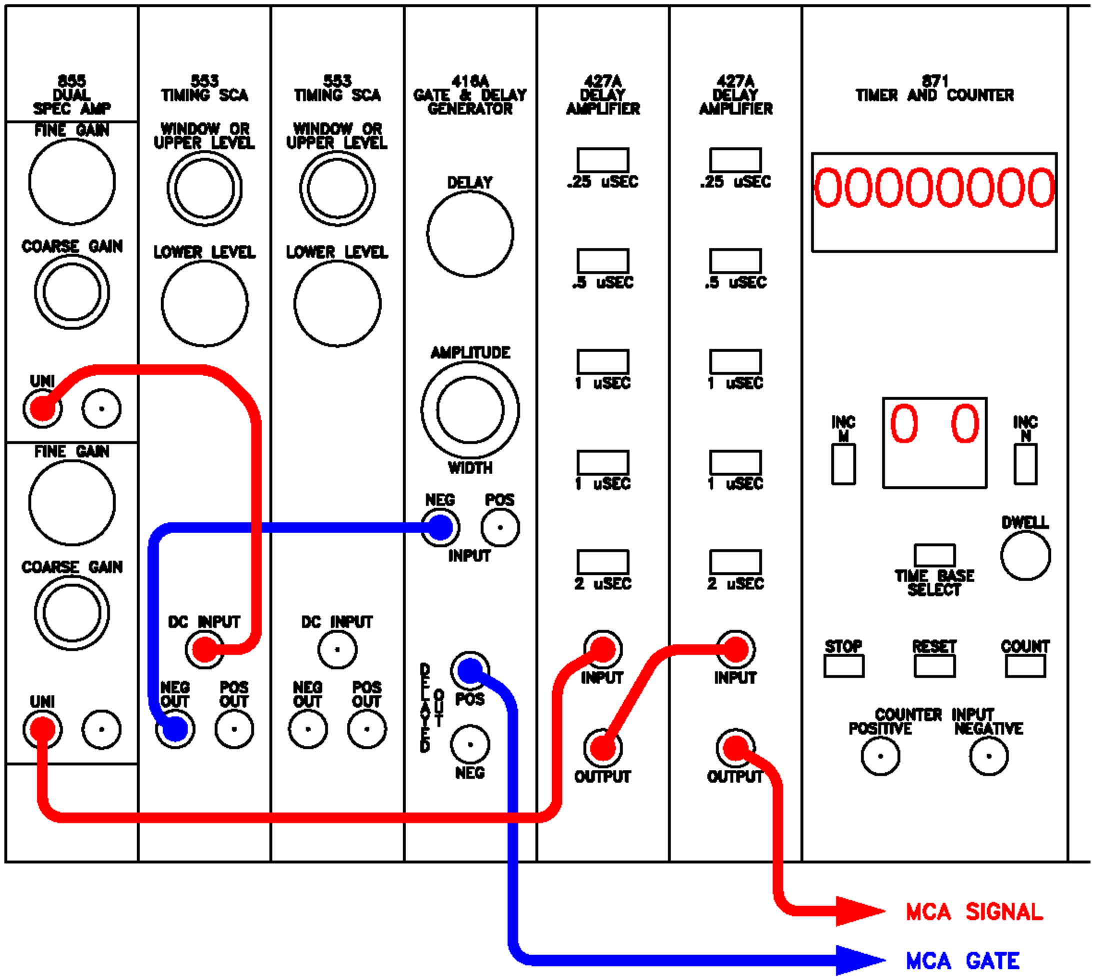

Figure 8. Wiring for measuring the recoil electron pulse height spectrum triggered by scattered gamma-rays. **Red** cables carry analog signals; **Blue** cables carry logic levels.

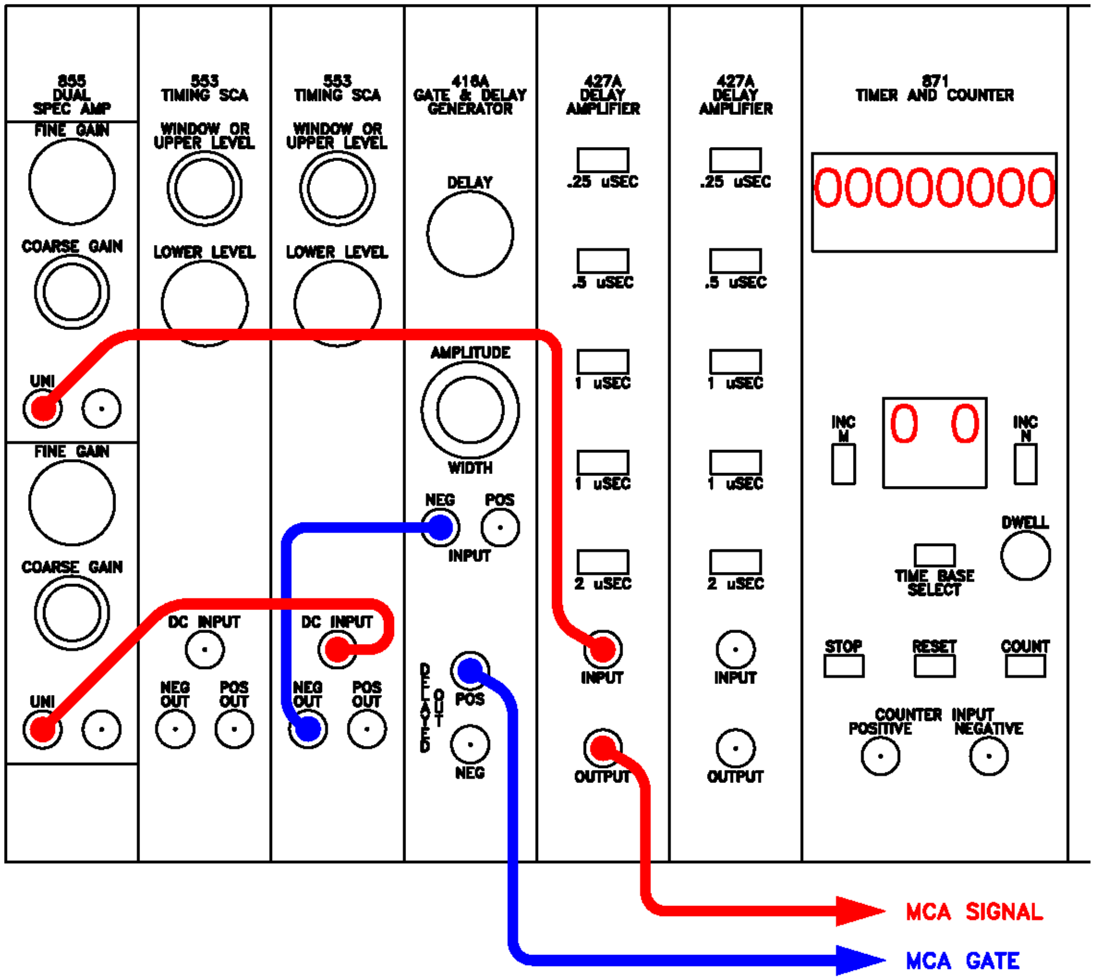

Figure 9. Wiring for measuring the scattered gamma-ray pulse height spectrum triggered by recoil electrons. **Red** cables carry analog signals; **Blue** cables carry logic levels.

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