COMPTON SCATTERING

INTRODUCTION

The scattering of X-rays by electrons was discovered by A. H. Compton in 1923. From careful measurements of the spectrum of scattered X-rays he showed that they suffered a shift to longer wavelengths. The size of the shift depended on the angle of scatter, and could be calculated by considering the process as a collision between a single photon and a single electron in which energy and momentum were conserved. The effect provides a striking demonstration of an electromagnetic wave behaving like a beam of "particles".

The main purpose of this experiment is to observe the effect of γ-rays being scattered by material, and to measure their energy shifts as a function of their scattering angles. The energy spectrum of the scattered γ-rays is measured using a sodium iodide crystal, in conjunction with a photomultiplier tube (PMT), to convert the γ-rays' energies to electrical pulses. The energy of the recoil electron is also measured using a scintillation counter that also serves as the target. Both the pulse height from the sodium iodide crystal and that from the scintillator can be recorded by an analog-to-digital converter or multichannel analyzer (MCA). A secondary objective is to familiarize you with some of the techniques of nuclear physics, such as scintillators, photomultiplier tubes (PMT's), oscilloscopes, fast coincidence circuits, and MCA's. (See Muon Lifetime write-up for PMT description.)

The relation between the energies of the incident and scattered γ-rays and the angle of scatter is readily obtained by considering the γ-rays to be photons, which strike free (i.e., unbound and at rest) electrons. The process is calculated by applying energy and momentum conservation to a collision between two elementary particles, a photon of energy $E$ and momentum $E/c$, and an electron of mass $m_e$ at rest. The photon is scattered through an angle $\theta$, and the scattered photon has energy $E'$ given by the following relation

$$\frac{1}{E'} - \frac{1}{E} = \frac{1 - \cos \theta}{m_e c^2}$$

where $m_e c^2$ is the rest-mass energy of the electron, which is equal to 0.511 MeV. Energy is conserved in the collision so the kinetic energy of the recoiling electron is just

$$K_e = E - E'$$

The derivation of Equation (1) is given in many texts. See, for example, Melissinos, 2nd edition, p. 369 ff. Note that the struck electron is treated as a free and stationary particle. This approximation is reasonable for a target composed of low-Z material, in which the binding energy of the struck electron is small in comparison to the shift in energy of the scattered photon. The plastic scintillator used as a target in this experiment is mostly polystyrene (CH).
METHOD

Refer to Figure 1. A beam of $\gamma$-rays from a radioactive source ($^{137}$Cs inside a lead collimator) is pointed at a plastic scintillator that is attached to PMT(1). The spectrum of $\gamma$'s from $^{137}$Cs has a prominent peak at 0.662 MeV. Some of the $\gamma$-rays are scattered and strike a second scintillator made of sodium iodide. This is a relatively dense material that absorbs all the energy of the photon and produces a pulse of light whose intensity is proportional to its energy. This pulse of light is collected by PMT(2), which produces an electrical pulse whose amplitude is proportional to the intensity of the light. The electrical pulses from both PMT’s after suitable amplification are "digitized" by an analog-to-digital converter (ADC) and read into a computer. [We use the AmpTek Pocket Multichannel Analyzers(MCA). See http://amptek.com/mca8000a.html and the X-ray Spectroscopy write-up for more.]

For convenience we will refer to the sodium iodide and PMT(2) as the $\gamma$ counter and the plastic scintillator and PMT(1) as the electron counter. The $\gamma$ scattering angle $\theta$ is restricted by the geometry. To reduce what would be a large background of direct (unscattered) $\gamma$'s reaching the $\gamma$ counter, lead shielding should be put between the source and $\gamma$-counter (Fig. 1). Use the lead brick with a hole in it as a collimator between the source and the electron counter. The long cylindrical lead shield should always be used in front of the $\gamma$ counter. The $\gamma$ pulse height (or energy) spectrum can be measured directly with this setup. To measure the electron pulse height spectrum we require that the pulse from PMT(2) be coincident in time with the pulse from PMT(1), which detects the light produced by the recoil electron in the plastic scintillator.

As shown in Figure 2, the pulses from both counters are fed to 485 Amplifiers and to discriminators, which output a logic pulse of fixed width and amplitude (−0.5 V) if the input amplitude exceeds a preset level. The discriminator outputs are fed to a coincidence circuit that gives an output only when pulses appear at both inputs simultaneously. Its output triggers a gate generator used to "gate" or enable the input to the MCA that digitize the pulses from the counter. The timing of the two inputs to the coincidence circuit must be adjusted by changing cable lengths so the electron pulses arrive at the same time as the $\gamma$ counter pulses. Ideally whenever a coincidence occurs it is produced by a $\gamma$ detected in the $\gamma$ counter and the recoil electron produced by the same $\gamma$. However, in practice many of the coincidences are produced by "accidental" or random coincidences between pulses from the two counters since the rates in the electron counter are high. The rate of accidentals can be reduced by keeping the $\gamma$ counter as close to the electron counter as possible (to increase the probability that the scattered $\gamma$ will be detected) and by setting up the electronics to obtain the best possible time resolution. The lead shielding near the electron counter and source also help by reducing $\gamma$'s going directly from the source to the $\gamma$ counter. Accidentals are especially serious near 0° because unscattered $\gamma$'s or $\gamma$'s which scatter through only a very small angle are detected in the $\gamma$ counter. Another problem when the $\gamma$ counter is at small angles is that the recoil electron has a low energy and the pulse may be too small to be detected. This makes it difficult to see coincident pulses when the $\gamma$ detector is at small angles.
SETUP

The setup should be pretty much complete, but you should work your way through the circuits and make sure you understand the functions of each module. Turn on the power to the NIM (Nuclear Instrumentation Module) crate and turn on the HV supply for the PMT’s. The HV supply should be set at 2000 V or the voltage indicated on the supply. The γ counter with its lead shield should be placed as close as possible to the target (e counter). Place the lead brick collimator close to the e counter. Recommended initial settings for the electronics are given in Fig. 2.

First observe the pulses from the electron and γ counters with an oscilloscope. For this you can use a low-activity “button” gamma source, such as 137Cs, placed right on the end of the counter. Because the pulses vary a lot in amplitude, it is best to use an analog oscilloscope with a bandwidth of at least 100 MHz, such as the Tektronix 2235. (Note that these are negative going.) Make sketches of these showing the width and amplitude range. With the oscilloscope, work your way through the amplifiers, discriminators, and coincidence circuit so you understand the analog and digital circuitry. Sketch the pulses and include these in your lab report. For this experiment the MCA must be "gated" so that it is only sensitive during the time we expect there to be a pulse. This is accomplished by forming a delayed pulse with the Gate Generator. Set the coincidence circuit to count "singles" from the γ counter (the A-AB-B switch toward "γ"). Use input channel 1 of the scope to look at the Gate Generator output; it should be a positive pulse about 2 µsec long and 4 V high. Connect Chan. 2 to the output of the γ 485 amplifier. If you trigger on Chan. 1 and observe both Chan. 1 and 2, you should see the shaped pulses from the γ counter with the positive lobe contained within the gate pulse.

Next get a pulse height spectrum for the γ counter and learn how to use the MCA. To do this, use a low-activity 137Cs source placed right on the counter, and feed the signal from the γ counter amplifier output to the MCA input. Accumulate a γ spectrum. It should show a prominent peak on the high end that corresponds to the 0.662 MeV peak in the 137Cs spectrum. (See Melissinos II, Fig. 8.21.) Adjust the gain controls on the 485 amplifier so this peak just fits into the high end of the MCA plot. Switch the source to the electron counter, set the coincidence circuit to electron counter singles, feed the amplified electron counter signal into the MCA, and set the electron counter gain controls so that the spectrum fits into the MCA range. Note that the electron peak will be very broad because scattered electrons have all energies up to ~0.66 MeV and the energy resolution of the plastic scintillator is not nearly as good as for sodium iodide. Record the electron and γ counter spectra. Also record the gain settings, the counter HV, the settings on the Gate Generator, and the MCA so you can reproduce these settings if data taking is interrupted.

COINCIDENCE TIMING

Now get the strong 137Cs source, which is housed, in a heavy cylindrical lead shield. Arrange as in Fig. 1. Note that there should be a lead brick just next to the e counter to prevent gammas going directly from the γ source to the γ counter. This is a strong source so it is a good idea to surround it with lead bricks as much as possible to reduce your exposure. Rotate the top so the source is aligned with...
the hole and set it against the opening in the lead brick collimator. For calibration it is convenient to put the $\gamma$ counter at about 45° relative to the $\gamma$'s from the source. Using a fast analog oscilloscope, try to observe "coincident" pulses from the two counters by triggering the scope on the $\gamma$ counter output and looking at the pulses from the electron counter. With the room lights out, you should be able to see a faint grouping of pulses that move if delay is added to the electron counter pulse by inserting a length of cable.

Put the two counters in coincidence (coincidence circuit switch on AB). Do a "delay curve" to verify that the $\gamma$ counter and electron pulses arrive at the coincidence circuit at the same time. Use the scaler to measure the rate of coincidences vs. delay. [Note that the scaler can be set up to run for a fixed length of time using the timer on Channel 1, so you just have to reset the scaler and hit the start button on the timer to take a point.] Vary the timing by adding lengths of cable to either counter between the discriminator and coincidence circuit. [The cable delay is about 1.5 ns/foot.] The "plateau" region, where the relative timing is correct, is fairly broad, so you can use 4 ns delay steps. Plot the rate vs. delay and leave the delay set to the center of the peak. The "tail" of the curve shows the substantial contribution of accidental coincidences to the overall rate.

**VERIFICATION OF THE COMPTON RELATIONS**

Measure the coincidence rate and accidental rates (by purposely mistiming the counters) for 6 or so angles from 5° to 115°. Reposition the lead shield between the source and $\gamma$ counter appropriately at each angle, but keep the radial separations between the counters and source constant. Plot the rate corrected for accidentals vs. angle.

Move the source farther away to get better angular resolution and get electron and $\gamma$ spectra at several angles. Use the lead shield to reduce accidentals. For each setting, collect a modest number of counts. You should see the energy of the electron increasing as that of the $\gamma$ decreases. Print out the plots and use the printout to check the prediction of Eq. (2).

Next switch the coincidence circuit to record singles from the $\gamma$ counter. Take $\gamma$ spectra at six or so angles between 10° and 115° and record the position of the energy-shifted 0.662 MeV peak as a function of angle. From these energy shift data determine the mass of the electron from Eq. (1). [See Melissinos, p. 382, 383]. Refer to the section on Energy Calibration below for suggestions on calibrating the energy scale.

Now take data to determine the angular dependence of the differential cross-section for Compton scattering for the range of angles $\theta = 5^\circ$ to $\theta = 115^\circ$. For this it is best to just use $\gamma$ singles rates measured with the scaler. Measure the rate as a function of angle without changing the spacing of the counters. Compare your angular dependence with the shape expected from the Thomson and Klein-Nishina formulas. [See Melissinos, Eq. 9.14 and Fig. 9.8]. The $\gamma$ counter is thicker than the one described in Melissinos and its efficiency can be taken as 100%.

It is difficult to measure absolute cross sections to check the Klein-Nishina formula, Melissinos II, Eqs. 9.13 and 8.22, because this requires a knowledge of the intensity of the source. However, the
following procedure should give a rough estimate. Rearrange the counters so that the \( \gamma \) counter is the same distance from the source as the electron counter was for the previous measurements. Keep the lead brick collimator in the same relative position as before. Measure the \( \gamma \) singles rate with the scaler to get an estimate of the number of \( \gamma \)'s per second, \( I_0 \), that were hitting the electron counter. The measured rate \( \Delta N/\Delta t \) at an angle \( \Theta \) is

\[
\Delta N/\Delta t = I_0 \left( d\sigma/d\Omega \right) n \Delta x \Delta \Omega
\]

where \( d\sigma/d\Omega \) is the differential cross section defined in Sect. 8.2.1 in Melissinos II; \( n \) is the number of electrons per unit volume in the scintillator target \( \approx 6 \times 10^{23} \text{ cm}^{-3} \), \( \Delta x \) is the thickness of the target (\( \approx 2.5 \text{ cm} \)), and \( \Delta \Omega \) is the solid angle subtended by the \( \gamma \) counter. The solid angle is \( \approx A/D^2 \) where \( A \) is the area of the \( \gamma \) counter (approx. 2.5 cm. diameter) and \( D \) is its distance from the target. Measure the \( \gamma \) counter singles rate at 90° with the electron counter as close as possible to the source and calculate \( d\sigma/d\Omega \) from Eq. 3. Compare with Klein-Nishina and Thomson predictions. Show your calculations.

**ENERGY CALIBRATION**

The 0.662 MeV peak from the \(^{137}\text{Cs}\) source is very conspicuous, especially in spectra taken at small angles, and is the primary "line" for calibration of the energy vs. MCA channel. However, the pulse height probably won’t extrapolate to \((0,0)\), and the pulse height may not be a linear function of \( \gamma \) or electron energy, especially for large energies. Therefore both counters should be calibrated at other energies. Other calibration points can be obtained with relatively weak radioactive sources such as \(^{57}\text{Co}\), \(^{22}\text{Na}\), and \(^{133}\text{Ba}\), that can be held close to the counters. The energies of the prominent peaks from these sources can be found in published tables, and in Melissinos, Sect. 9.2. The pulse height spectrum for electrons from Compton scattering won’t give a peak, but you can get an estimate by assuming the largest pulses correspond to the highest-energy \( \gamma \) peak from the source.

**QUESTIONS**

1. How do the accidental rates compare with the "real" rates at large \( \gamma \) angles? At small angles? Why would you expect the real/accidental ratio to go down at small angles?
2. Do your plots of electron energy vs. \( \gamma \) energy show the behavior expected from Eq. 2? What kind of variation is expected? Discuss why the pulse heights might not show the expected behavior.
3. Do your \( \gamma \) energy vs. angle data agree with the expected behavior? Discuss reasons why it might not.
4. Does the differential cross section estimated from your data at 90° agree with that predicted from the Klein-Nishina formula? The Thomas formula? Discuss.
5. Compare the pulses from the \( \gamma \) counter and those from the electron counter. Can you explain why the pulse widths are so different?
6. Use Eq. 1 and your \( \gamma \) energy vs. angle data to determine the electron mass.
Figure 1 – Schematic of the setup

Figure 2 – The electronics. Note that the inputs of the 485 amplifiers and MCA are high impedance while most other inputs are 50 ohm to properly terminate the coaxial cables. At least one of each output pairs of the T105s and C104 modules must be terminated in 50 ohm for them to work properly. The pocket MCA input is positive-going, 0 to 10 V. The gate is a 4 V positive pulse; it should span the peak in the input pulse and extend >1 µsec after the peak. The small connectors on the MCA are called "LEMO" connectors. The larger twist on ones are "BNC".