Cosmic Microwave Background Radiation

Carl W. Akerlof
April 7, 2013

Notes:

Dry ice sublimation temperature:	194.65° K
Isopropyl alcohol freezing point:	184.65° K
LNA operating voltage:	18.0 v

The terrestrial coordinates for the Block M on the Diag are 42º 16.621′ N, 83º 44.292′ W, 280 m altitude.

The core of this experiment is a series of measurements of the black body emission of objects at various temperatures at a frequency of 12 GHz.

The Planck spectral distribution function is given by:

$$\frac{dI}{df} = \frac{2\pi h\nu^3}{c^2}\frac{1}{e^{h\nu/kT} - 1}$$

Setting $h\nu = kT_\nu$, a frequency of 12 GHz is equivalent to a temperature of 0.576° K. This is small relative to the three temperatures that are important for this experiment, $T_{CNBR} = 2.725°\text{ K}$, $T_{LN2} \cong 77.348°\text{ K}$ and $T_{room} \cong 293.15°\text{ K}$. Thus, the spectral distribution can be approximated as an expansion in $x = h\nu / kT$:

$$\frac{dI}{df} = \frac{2\pi\nu^2 kT}{c^2}\left(1 - \frac{1}{2}x + \frac{1}{12}x^2 + \ldots\right)$$

For the temperatures of interest in this experiment, the terms in *x* can be ignored and the spectral intensity is simply proportion to *T*.

The temperatures of the two thermal reference loads are estimated in two different ways. For the room temperature load, the temperature can be directly determined by the yellow “lollipop” thermometer. For the liquid nitrogen load, the temperature is a smoothly varying function of the ambient atmospheric pressure:

$$T = 77.348 + 8.68727\log_e\left(P / P_o\right)$$

where P_o is one atmosphere in whatever units that are used to measure *P*.

The voltage output of the 12 GHz LNA plus diode detector is proportional to the sum of two terms, a characteristic amplifier noise temperature, T_{amp}, and the black body thermal radiation, T. Thus,

$$V = c \cdot \left(T + T_{amp} \right)$$

With calibration measurements, $(T_1,\ V_1)$, at room temperature and $(T_2,\ V_2)$ at the liquid nitrogen boiling point, T_{amp} has the following value:

$$T_{amp} = \frac{T_1 V_2 - T_2 V_1}{V_1 - V_2}$$

It follows that for additional measurements of a receiver output voltage, V, the corresponding temperature is given by:

$$T = \frac{\left(V - V_2\right) T_1 + \left(V_1 - V\right) T_2}{V_1 - V_2}$$

Assuming uncorrelated measurement errors of $\sigma_{T_{12}}$ and σ_V for load temperatures and receiver voltages, the error in measured temperature can be computed by standard error proportion techniques:

$$\sigma_T^2 = \frac{V_1^2 + V_2^2 - 2\left(V_1 + V_2 - V\right)V}{\left(V_1 - V_2\right)^2} \sigma_{T_{12}}^2 + \frac{2\left(T_1 - T_2\right)^2 \left(V_1^2 - V_1 V_2 + V_2^2 - \left(V_1 + V_2 - V\right)V\right)}{\left(V_1 - V_2\right)^4} \sigma_V^2$$

The critical measurements for this experiment involve observations of the sky from the roof of Angell Hall. The alti-az mount is designed for increments of 5° in zenith angle and 10° in azimuth. The azimuth setting ring should be placed so that one position is due south, aligned appropriately with the longitudinal axis of the building. You will need to make 12 measurements of receiver output for zenith angles from 0° to 55°, repeated for azimuth angles taken on 30° intervals. Repeated calibration with the room temperature and liquid nitrogen thermal reference loads is required to remove systematic drifts.

To reduce the data, take each set of 12 measurements at fixed zenith angle. A robust measure of average in this case is the median of the 12 values. A useful estimate of the error can be obtained from the interquartile difference (see statistics notes). In this particular case, an estimate of the standard deviation can be obtained from:

$$\sigma \simeq 0.37065055\left(T_{10} + T_9 - T_4 - T_3\right)$$

where the sky temperatures, $\{T_i\}$ have been ordered by increasing value. These should be compared with the error analysis described above.

A plot of the sky temperature vs. secant(θ_{zenith}) should suggest a straight line that can be extrapolated to zero. The intercept corresponding to zero path length in the atmospheric is your estimate of the astrophysical contribution, the CMBR. Estimate the error for this value based on the errors of the values that contribute to the computation.

Two other measurements can be obtained with this equipment. There are two point sources of 12 GHz radiation that can be sensed. One is the Sun and the second is a geosynchronous satellite due south. A lens and screen has been attached to the receiver housing to align the Sun with the receiver axis. The small holes in the frosted screen are drilled on a 5° grid. The geosynchronous satellite can be easily found once you estimate its zenith angle. That can be estimated from the following parameters:

G	6.67384×10^{11}	$m^3\ kg^{-1}\ s^{-2}$
M_{Earth}	5.9736×10^{24}	kg
R_{Earth}	6371	km
$t_{sidereal}$	86164.1	s
$\vartheta_{latitude\ UM}$	42.277	°

Table 1. Orbital parameters

A Slightly Better Analysis Procedure

The data analysis method described above is slightly misleading because it tends to confuse temperature with spectral intensity, the physical variable that is actually being directly measured. In particular, when one points the receiver at the sky, the radiation intensity is comparable to a black body at 40° K. This, of course, is not a real temperature. The origin of this 12 Ghz radiation is the atmosphere with a scale height of 8 km. Its actual temperature is of the order of 250° K but it is highly transparent at these frequencies and therefore the intensity we observe is correspondingly lower. What we do observe is proportional to the path length through the medium and therefore proportional to $\sec(\theta_{zenith})$. We also would like to avoid the numerical approximation that $e^x - 1 \simeq x$ if this is not absolutely necessary.

The starting point of this analysis is to track a quantity that is directly related to intensity, not temperature:

$$u \equiv \frac{1}{e^{h\nu/kT} - 1} = \frac{1}{e^{T_\nu/T} - 1}$$

and T can be computed from:

$$T = \frac{T_\nu}{\log\left(1+1/u\right)}$$

From a set of calibration measurements, $u_1,\ V_1,\ u_2,\ V_2$ and an observation, $V,$ one can obtain an estimate of *u*:

$$u = \frac{\left(V-V_2\right)u_1 + \left(V_1-V\right)u_2}{V_1-V_2}$$

with a variance:

$$\sigma_u^2 = \frac{2\left(u_1-u_2\right)^2\left(V_1^2-V_1V_2+V_2^2-\left(V_1+V_2-V\right)V\right)}{\left(V_1-V_2\right)^4}\sigma_V^2$$

$$+\frac{\left(u_1(1+u_1)(V-V_2)/T_1^2\right)^2+\left(u_2(1+u_2)(V-V_1)/T_2^2\right)^2}{\left(V_1-V_2\right)^2}T_\nu^2\sigma_{T_{12}}^2$$

These intensity amplitudes should be plotted as a function of $\sec\left(\theta_{zenith}\right)$ and the intercept and error determined by the usual techniques for linear regression. The CMBR temperature can then be determined by the formula given above and its standard deviation is:

$$\sigma_T = \frac{T_\nu}{u\left(1+u\right)\log^2\left(1+\frac{1}{u}\right)}\sigma_u$$

1.	tea cart
2.	two thermal reference loads
3.	“lollipop” thermometer
4.	two liquid nitrogen buckets
5.	12 GHz receiver assembly
6.	receiver azimuth ring
7.	hex wrench for azimuth ring
8.	Agilent E3632A DC power supply
9.	Agilent 34401A multimeter
10.	two coax cables

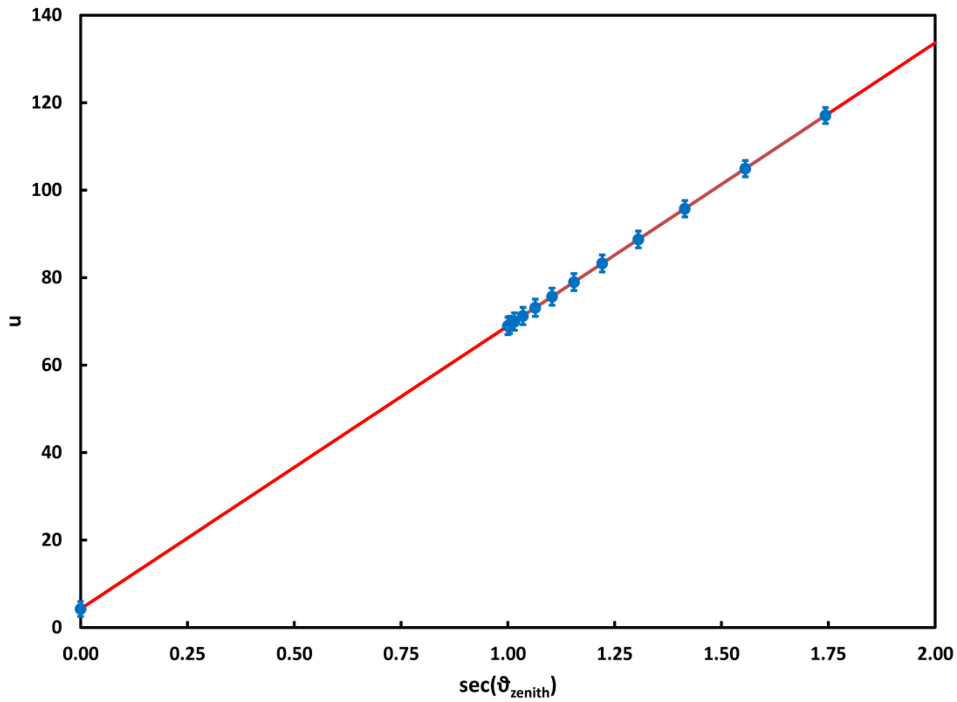
Table 2. Equipment for Angell Hall

A	B	C	D
295°	300°	305°	310°
315°	320°	325°	330°
335°	340°	345°	350°
355°	0°	5°	10°
15°	20°	25°	30°
35°	40°	45°	50°
55°	60°	65°	70°
75°	80°	85°	90°
95°	100°	105°	110°
115°	120°	125°	130°
135°	140°	145°	150°
155°	160°	165°	170°
175°	180°	185°	190°
195°	200°	205°	210°
215°	220°	225°	230°
235°	240°	245°	250°
255°	260°	265°	270°
275°	280°	285°	290°

Table 3. Zenith angles for receiver mount. The outer and inner index plates are drilled on 15° and 20° centers, respectively.

1	2	3	4
0°	10°	20°	30°
40°	50°	60°	70°
80°	90°	100°	110°
120°	130°	140°	150°
160°	170°	180°	190°
200°	210°	220°	230°
240°	250°	260°	270°
280°	290°	300°	310°
320°	330°	340°	350°

Table 4. Azimuth angles for receiver mount, assuming proper ring alignment. The upper and lower index plates are drilled on 30° and 40° centers, respectively.



140
120
100
80
60
40
20
0
u
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
sec(ϑzenith)