Problem 2: A Man Waking on a Platform

A man of 100 kg is standing on the rim of a platform of mass 1000 kg that is rotating at an angular speed 2 rad/s. The platform acts like a solid disk with radius 10 m and has perfect bearings. (The moment of inertia of the platform is I_p=1/2MR^2.) After enjoying the view for a minute or two, the man walks slowly towards the center of the platform.

1. What is the moment of inertia of the man, I_{mi}, when he is standing on the rim?

$$I_{mi} = mR^2 = 100\times10^2 = 10^4 \text{ (kg}\cdot\text{m}^2\text{)}$$

2. What is the total moment of inertia of the system (platform + man), I_i, when the man is standing on the rim?

$$I_i = I_{mi} + I_p = 10^4 + \frac{1}{2}\times1000\times10^2 = 6\times10^4 \text{ (kg}\cdot\text{m}^2\text{)}$$

3. What is the total angular momentum of the system, $\mathbf{L}_i$, when the man is standing on the rim?

$$L_i = I\omega_o = (6\times10^4)(2) = 1.2\times10^5 \text{ (kg}\cdot{}^{\text{m}^2}\!/_{\text{s}}\text{)}$$

4. What is the momentum of inertia of the man, I_{mf}, when he is halfway from the rim to the center?

$$I_{mf} = m\left(\frac{1}{2}R\right)^2 = 100\times5^2 = 2.5\times10^3 \text{ (kg}\cdot\text{m}^2\text{)}$$

5. What is the total momentum of inertia of the system, I_f, when the man is halfway from the rim to the center?

$$I_i = I_{mf} + I_p = 2.5\times10^3 + \frac{1}{2}\times1000\times10^2 = 5.25\times10^4 \text{ (kg}\cdot\text{m}^2\text{)}$$

6. If the angular speed of the system is ω_f when the man is halfway from the rim to the center, what is the total angular momentum of the system, $\mathbf{L}_f$, at this moment?

$$L_f = I_f \omega_f$$

7. The angular momentum of the system is conserved. Why?

Because the net external torque is zero

8. With the information you obtained in 7, solve ω_f.

$$L_f = L_i$$
$$\Rightarrow I_f \omega_f = 1.20 \times 10^5$$
$$\Rightarrow \omega_f = \frac{1.20 \times 10^5}{5.25 \times 10^4} = 2.29 \text{ (rad/s)}$$

9. What is the initial rotation kinetic energy of the system, KE_i?

$$KE_i = \frac{1}{2} I_i \omega_o^2 = \frac{1}{2}(6 \times 10^4)(2)^2 = 1.2 \times 10^5 \text{ (J)}$$

10. What is the final rotation kinetic energy of the system, KE_f?

$$KE_f = \frac{1}{2} I_f \omega_f^2 = \frac{1}{2}(5.25 \times 10^4)(2.29)^2 = 1.37 \times 10^5 \text{ (J)}$$

11. How much work does the man do in moving halfway to the center?

$$W_{net} = \Delta KE_f = (1.37 - 1.2) \times 10^5 = 1.7 \times 10^4 \text{ (J)}$$