$\boxed{3\text{-}21}$

THIS IS LECTURE #3 CONCEPT TEST #4 WITH CONSTANT-VELOCITY X MOTION SUPERPOSED ON THE y MOTION. BOTH TANKS ARE LAUNCHED WITH SPEED v_0 AT AN ANGLE ϕ EITHER ABOVE OR BELOW THE HORIZONTAL. THE INITIAL LAUNCH VELOCITY VECTORS ARE:

$$\vec{v}_{up}\left[+v_0\cos\phi,\ +v_0\sin\phi\right]$$

$$\vec{v}_{DOWN}\left[+v_0\cos\phi,\ -v_0\sin\phi\right]$$

BOTH TANKS ARE LAUNCHED AT A HEIGHT H ABOVE THE GROUND. THE UPWARD-LAUNCHED TANK WILL RISE TO AN ADDITIONAL HEight

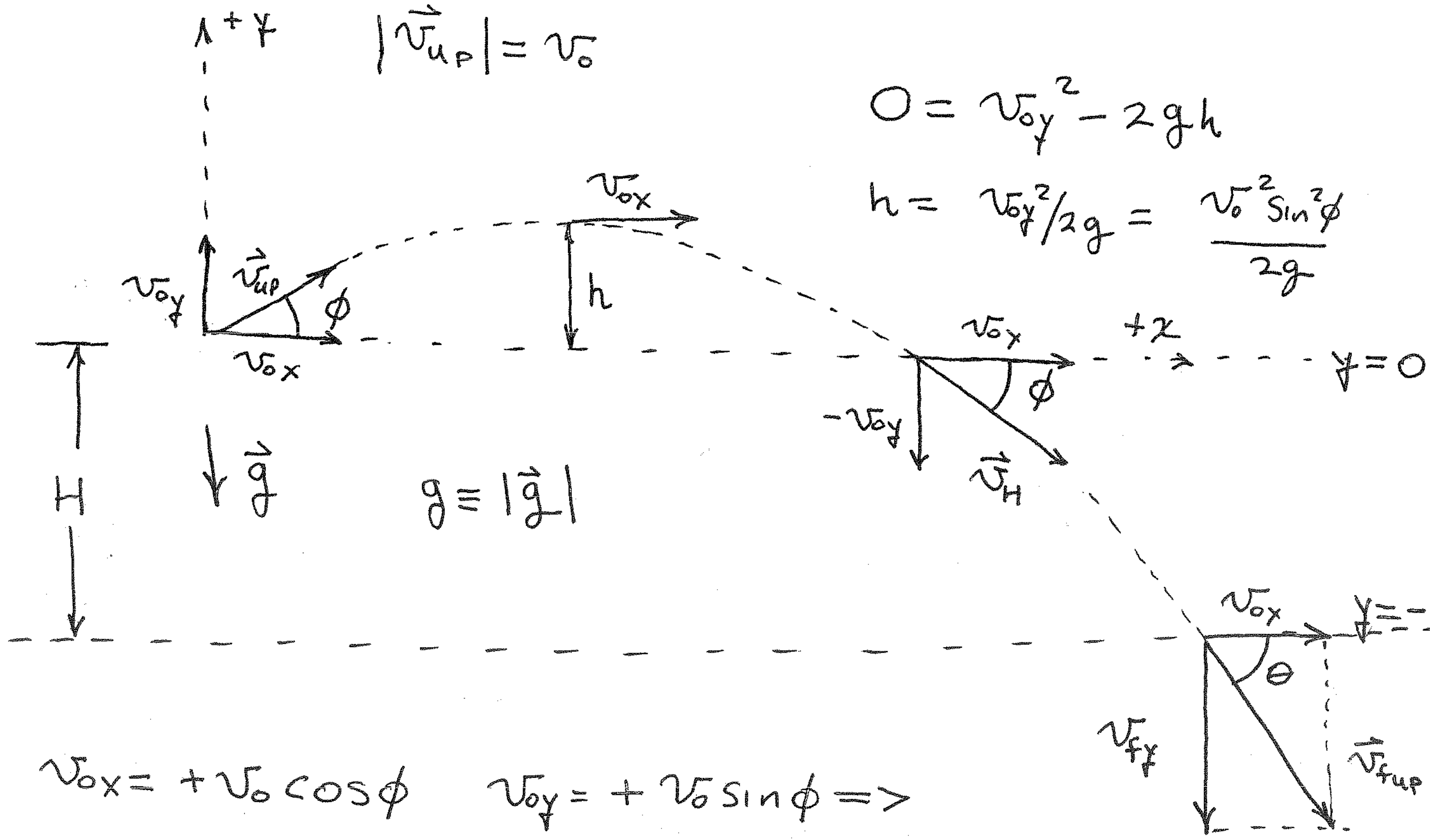
$$h = \frac{v_{0y}^2}{2g} = \frac{v_0^2\sin^2\phi}{2g}$$

AND THEN RETURN TO THE ORIGINAL HEIGHT H WHERE ITS VELOCITY VECTOR WILL BE

$$\vec{v}_H\left[+v_0\cos\phi,\ -v_0\sin\phi\right] \equiv \vec{v}_{DOWN}$$

FROM THIS POINT ON, THE MOTION OF THE UPWARD-LAUNCHED TANK WILL BE IDENTICAL TO THE MOTION OF THE DOWNWARD-LAUNCHED TANK AND THE TWO TANKS WILL IMPACT WITH IDENTICAL FINAL VELOCITY VECTORS.

UPWARD-LAUNCHED TANK



$$0 = v_{oy}^2 - 2gh$$

$$h = v_{oy}^2/2g = \frac{v_o^2 \sin^2\phi}{2g}$$

$v_{ox} = +v_o \cos\phi \qquad v_{oy} = +v_o \sin\phi \Rightarrow$

$\vec{v}_{up}\,[+v_o\cos\phi\,,\ +v_o\sin\phi]$

$\vec{v}_H\,[+v_o\cos\phi\,,\ -v_o\sin\phi]$ $\quad \boxed{\vec{v}_{fup}\left[+v_o\cos\phi,\ -(v_o^2\sin^2\phi + 2gH)^{1/2}\right]}$

$$v_{fy}^2 = [+v_{oy}]^2 - 2g[-H] = [+v_o \sin\phi]^2 + 2gH$$

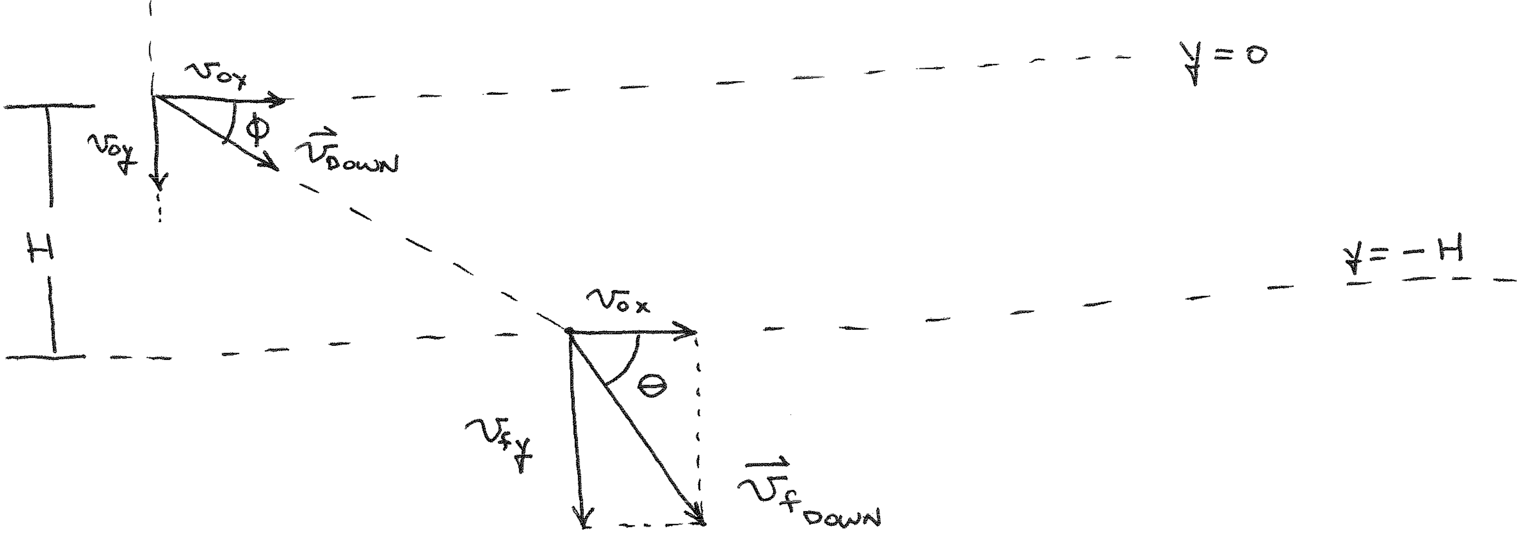
$$\boxed{v_{fy}^2 = v_o^2 \sin^2\phi + 2gH}$$

DOWNWARD-LAUNCHED TANK

$\downarrow \vec{g}$

$$|\vec{v}_{DOWN}| \equiv v_0$$

$$g \equiv |\vec{g}|$$



$$\vec{v}_{DOWN}\left[+v_0\cos\phi,\ -v_0\sin\phi\right]$$

ONLY DIFFERENCE BETWEEN $\vec{v}_{UP}$ AND $\vec{v}_{DOWN}$

THE SIGN SQUARES OUT HERE

$$v_{fy}^2 = \left[\pm v_0\sin\phi\right]^2 - 2g(-H)$$

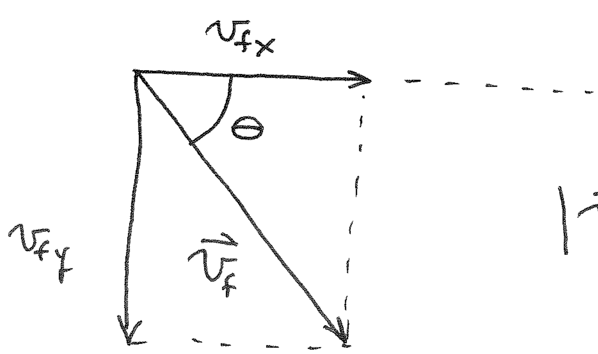
$$\boxed{v_{fy}^2 = v_0^2\sin^2\phi + 2gH} \implies$$

$$\boxed{\vec{v}_{f_{DOWN}}\left[+v_0\cos\phi,\ -\left(v_0^2\sin^2\phi + 2gH\right)^{1/2}\right] = \vec{v}_{f_{UP}}}$$

AND WE HAVE PROVEN THAT BOTH TANKS IMPACT WITH IDENTICAL FINAL VELOCITY VECTORS

$$\vec{v}_f \left[+ v_0 \cos\phi , - (v_0^2 \sin^2\phi + 2gH)^{1/2} \right] = \vec{v}_{f_{UP}} = \vec{v}_{f_{DOWN}}$$

WE CAN GET THE MAGNITUDE AND DIRECTION OF THE FINAL VELOCITY VECTOR FROM:



$$|\vec{v}_f| = \sqrt{v_{fx}^2 + v_{fy}^2}$$

$$\theta = \tan^{-1} \left[\frac{|v_{fy}|}{|v_{fx}|} \right]$$

WHERE θ IS AN ANGLE BELOW THE HORIZONTAL AS SHOWN. NOW IT REMAINS ONLY TO PUT IN THE NUMBERS...

$v_0 = 135 \text{ m/s}$ $\phi = 15°$ $H = 2.00 \text{ km} = 2\times10^3 \text{ m}$

$v_{fx} = (135 \text{ m/s}) \cos 15° = 130.4 \text{ m/s}$

$v_{fy} = -\left([135 \text{ m/s}]^2 \sin^2 15° + 2(9.8 \text{ m/s}^2)(2\times10^3 \text{ m}) \right)^{1/2} = 201.0 \text{ m/s}$

$$\boxed{\vec{v}_f [130.4 \text{ m/s}, -201.0 \text{ m/s}]}$$

$|\vec{v}_f| = \left[(130.4 \text{ m/s})^2 + (-201.0 \text{ m/s})^2 \right]^{1/2} = 239.6 \text{ m/s}$

$\theta = \tan^{-1} \left[\frac{201.0 (\text{m/s})}{130.4 (\text{m/s})} \right] = 57°$ BELOW HORIZONTAL

THIS IS THE IMPACT VELOCITY FOR BOTH TANKS.