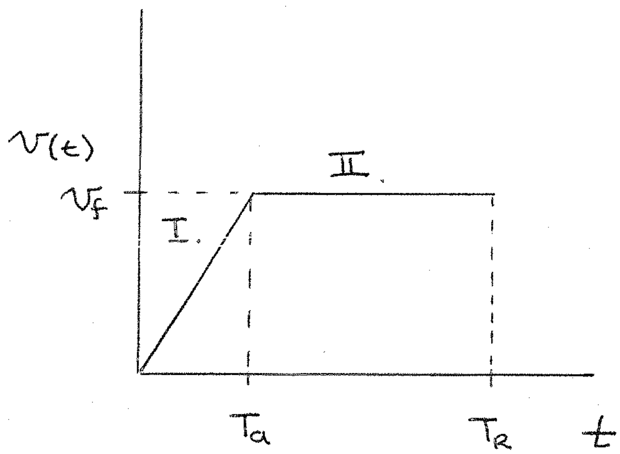
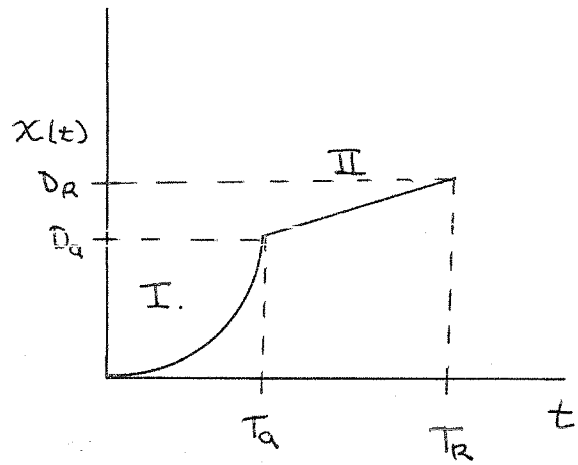
CH2 #36





MOTION IN TWO PARTS:

I. ACCELERATE FROM REST TO FINAL VELOCITY v_f

II. CONTINUE WITH CONSTANT VELOCITY v_f TO FINISH RACE

$a \equiv$ ACCELERATION FOR PART I GIVEN

$D_R \equiv$ TOTAL DISTANCE FOR RACE GIVEN

$T_R \equiv$ TOTAL TIME TO RUN RACE GIVEN

$T_a \equiv$ TIME TO ACCELERATE TO v_f IN PART I. UNKNOWN

$D_a \equiv$ DISTANCE TO ACCELERATE TO v_f IN PART I. FIND

$\boxed{\text{CH2 \#36}}$

PART I: $\boxed{T_a \geq t \geq 0}$ ACCELERATE FROM REST

TO FINAL VELOCITY v_f IN A TIME T_a AND A DISTANCE $D_a \Longrightarrow$

STARTS AT REST

$$x(t) = \cancel{x_0}^{\,0} + \cancel{v_0}^{\,0}\, t + \frac{1}{2} a t^2$$

STARTS at REST

$$v(t) = \cancel{v_0}^{\,0} + at$$

Define START POSITION $x \equiv 0$

$$\boxed{x(t) = \frac{1}{2} a t^2}$$

$$\boxed{v(t) = at}$$

$$x(T_a) \equiv D_a = \frac{1}{2} a T_a^2$$

$$v(T_a) \equiv v_f = a T_a$$

$$\boxed{D_a = \frac{1}{2} a T_a^2}$$

$$\boxed{v_f = a T_a}$$

PART II: $\boxed{T_R \geq t \geq T_a}$ BEGIN at $X = D_a$ And

MOVE WITH VELOCITY v_f FOR TIME REMAINING IN RACE $\equiv (T_R - T_a)$

CONSTANT VELOCITY FOR PART II.

$$x(T_R) = x_0 + v_f\,(T_R - T_a)$$

$$D_R = D_a + v_f\,(T_R - T_a)$$

TIME REMAINING IN RACE After $t = T_a$

INITIAL POSITION FOR PART II

FINAL VELOCITY FROM PART I

CH2 #36

SUBSTITUTE $\underline{D_a = \frac{1}{2} a T_a^2}$ and $\underline{V_f = a T_a}$

$$D_R = \overbrace{\frac{1}{2} a T_a^2}^{D_a} + (\overbrace{a T_a}^{V_f})(T_R - T_a)$$

QUADRATIC IN THE UNKNOWN TIME T_a. a, D_R & T_R ARE ALL KNOWN CONSTANTS $\Longrightarrow$ SOLVE QUADRATIC FOR T_a $\Longrightarrow$

$$D_R = \frac{1}{2} a T_a^2 + [a T_R] T_a - a T_a^2$$

$$D_R = -\frac{1}{2} a T_a^2 + [a T_R] T_a$$

$$-2 D_R = a T_a^2 - 2 [a T_R] T_a$$

$$\boxed{a T_a^2 - 2 [a T_R] T_a + 2 D_R \equiv 0}$$

$$A T_a^2 + B T_a + C = 0$$

$A = a$

$B = -2 [a T_R]$

$C = 2 D_R$

SOLVE FOR T_a WITH QUADRATIC FORMULA

$$T_a = \frac{1}{2a}\left[+2(a T_R) \pm \sqrt{[2a T_R]^2 - 4a(2D_R)}\right]$$

$$T_a = \frac{1}{2a}\left[2aT_R \pm \sqrt{[2aT_R]^2 - 8aD_R}\right]$$

$$T_a = \left[T_R \pm \frac{1}{2a}[2a]\sqrt{T_R^2 - 2\frac{D_R}{a}}\right]$$

$$\boxed{T_a = \left[T_R \pm \sqrt{T_R^2 - 2\left[\frac{D_R}{a}\right]}\right]}$$

$a = 3.59\ m/s^2 \qquad T_R = 7.97\,s \qquad D_R = 50\,m$

SINCE $T_a < T_R \Rightarrow$ PICK $(-)$ ROOT $\Rightarrow$

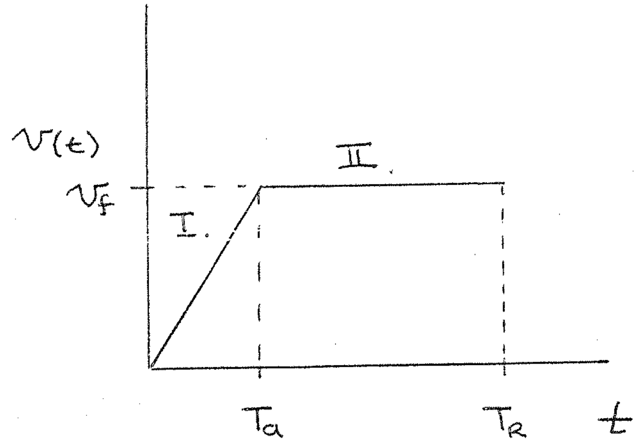
$$T_a = 7.97\,s - \sqrt{(7.97s)^2 - 2\left[\frac{50m}{3.59\ m/s^2}\right]}$$

$$T_a = 7.97s - 5.972\,s = \underline{1.9979\,s} \checkmark$$

FROM I.

$$D_a = \frac{1}{2}aT_a^2 = \frac{1}{2}[3.59\ m/s^2][1.9979s]^2$$

$$\boxed{D_a = 7.1650\,m} \checkmark$$



WE CAN ALSO SOLVE FOR THE ACCELERATION TIME T_a BY CALCULATING THE DISPLACEMENT FROM THE ORIGIN AS THE AREA UNDER THE v VS. t GRAPH:

$$D_R = \underbrace{\frac{1}{2}(v_f + 0)T_a}_{\text{AREA I}} + \underbrace{v_f(T_R - T_a)}_{\text{AREA II}} \qquad v_f = aT_a \Rightarrow$$

$$\boxed{D_R = \frac{1}{2}(aT_a)T_a + aT_a(T_R - T_a)}$$