I Still Don't Understand that "Shoot the Bear" Demo

The situation is shown in Figure 1.

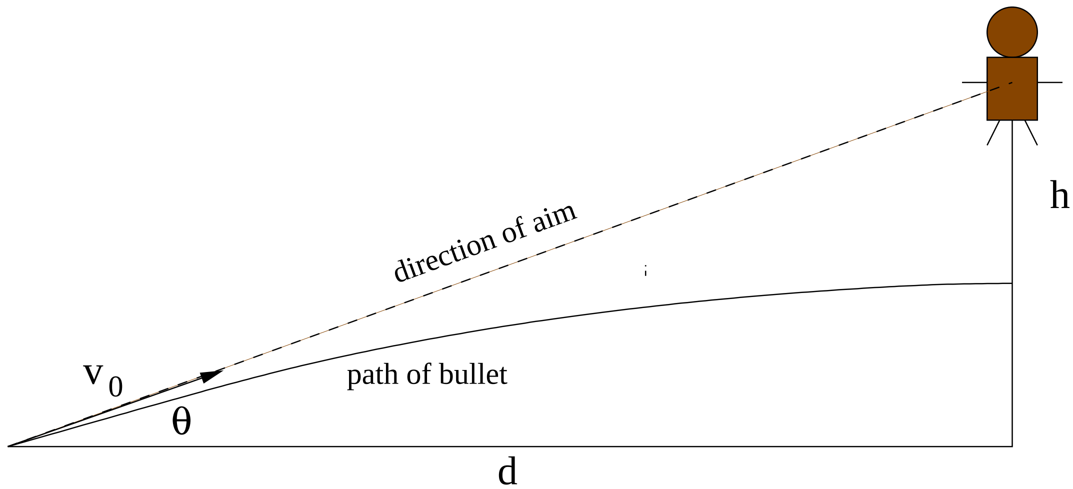

Figure 1: The "shoot the bear" demo. The gun is aimed directly at the bear. At the instant the bullet is fired with speed v_0, the bear drops from a height h.

We can phrase the problem mathematically as follows: "What is the vertical position of the bear, and the vertical position of the bullet, when the bullet has moved through the horizontal distance d?" If the two vertical positions are equal, the bullet hits the bear.

To find the vertical positions of either the bullet or the bear, we need to know how long it takes the bullet to travel the horizontal distance d. This is given by

$$t = \frac{d}{v_{0,x}} = \frac{d}{v_0 \cos\theta} \tag{1}$$

Now what are the vertical positions of the bear and the bullet at this time? Well, the bear starts from height h at $t = 0$ and drops from rest, so

$$y_{\text{bear}} = h - \frac{1}{2}gt^2. \tag{2}$$

The bullet starts from $y = 0$ and has a vertical velocity of $v_{0,y}$, so

$$y_{\text{bullet}} = v_{0,y}t - \frac{1}{2}gt^2 = (v_0 \sin\theta)t - \frac{1}{2}gt^2. \tag{3}$$

We need these two vertical positions to be equal at time $t = d/v_0 \cos\theta$ when the bullet has traveled the horizontal distance to the bear. Comparing Equation (2) to Equation (3), we see that they differ in the first term; the $-\frac{1}{2}gt^2$ terms are common to both. So the vertical positions will be equal if and only if $(v_0 \sin\theta)t = h$. Does it? Let's substitute our expression for t from Equation (1):

$$\begin{aligned}(v_0 \sin\theta)t &= v_0 \sin\theta \cdot \frac{d}{v_0 \cos\theta} \\ &= d\frac{\sin\theta}{\cos\theta} \\ &= d\tan\theta.\end{aligned}$$

But look again at Figure 1:

$$\tan\theta = \frac{(\text{opp})}{(\text{adj})} = \frac{h}{d},$$

so

$$(v_0 \sin\theta)t = d\tan\theta = d \cdot \frac{h}{d} = h.$$

Therefore the vertical positions of the bullet and the bear are equal at time t, so the poor critter meets his end. Notice that v_0 canceled out, so this result is independent of the muzzle speed of the gun—provided, of course, that this speed is large enough for the bullet to reach the bear.

Here's a helpful way to think about this experiment. Imagine that we could turn gravity off, i.e. set $g = 0$. In that case, the bullet travels along the straight line shown in the figure, and the bear does not fall anywhere when it lets go. So we obviously hit the bear then, for any value of v_0. When we turn gravity back on, both the bear and the bullet will "fall". Since all objects fall at the same rate, the bullet still intercepts the bear.

Perhaps it's confusing to think of the bullet as "falling" when it might be traveling up the whole time. Think of its "fall" as the difference between where it would have hit without gravity (the initial position of the bear, at height h), and the lower position where it actually hits. In that sense, the bear and the bullet both "fall" through the same distance.

Answers to frequently-asked questions:

- **If the bear had not let go, the bullet would have missed, right?** Yes–unless v_0 is large enough that the bullet "falls" by less than the height of the bear. (In that case, if I aim at his nose, I might hit him in the belly.)
- **But for a real gun, when the bullet travels in a straight line, it would be better for the bear to let go, right?** No. Bullets are projectiles too, and do not travel in straight lines. They "fall" like other projectiles. We are not usually aware of this fact because most guns are fired at rather nearby targets, and the bullets reach the target very quickly, so they do not have much time to fall. Over a typical 50 meter range, a bullet might fall only a millimeter or two compared to a straight line path. (Long-range sharpshooters do need to take the fall of the bullet into account.) If I had fired a real gun, the bullet would have reached the bear in a tiny fraction of a second, and the bear would have hardly fallen any distance before getting hit.
- **Is the Society for the Prevention of Cruelty to Animals aware that you do this demo?** This is a specially trained stunt bear. No actual bears were harmed in the performance of this demonstration.