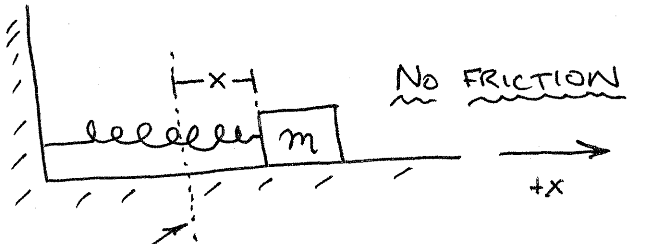
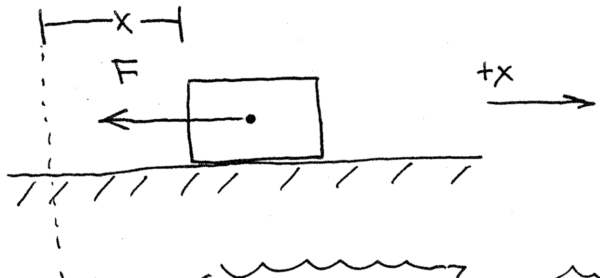
Discussion #19

Springs, Simple Harmonic Motion, and Elastic Potential Energy



Unstretched position of spring's end

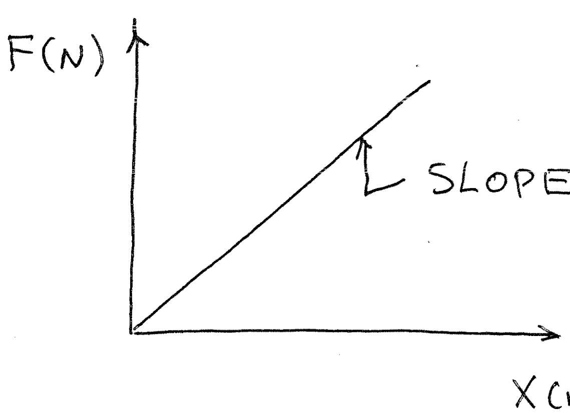


$$\boxed{F = -kx}$$

Hooke's Law: Force is proportional to x and in the opposite (−x) direction ≡ "Restoring Force"

k ≡ proportionality constant between spring displacement (x) and force (F)

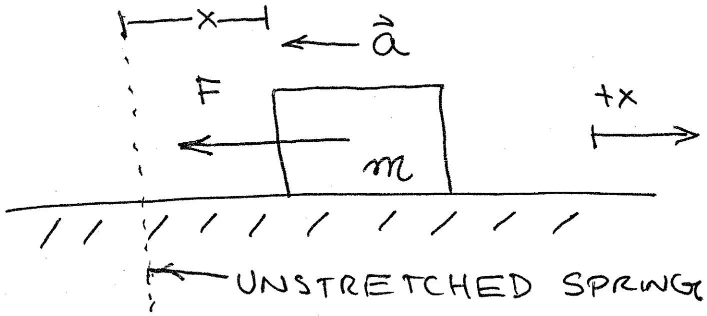
$$k \equiv \text{"Spring Constant"} = \frac{\text{Force}}{\text{Length}} \equiv \frac{N}{m}$$



$$\text{Slope} = \frac{\Delta F}{\Delta x} = k$$

SIMPLE HARMONIC MOTION (SHM)

APPLY NEWTON'S SECOND LAW:



$$F = -kx$$

$$F = ma$$

$$ma = -kx$$

$$\boxed{a = -\frac{k}{m}x} \quad \text{I}$$

a, the acceleration, is the second derivitive of the position x, so we have a differential equation describing the motion x(t)

$$\boxed{\frac{d^2 x(t)}{dt^2} = -\frac{k}{m}x} \quad \text{II}$$

This has the general form:

$$\boxed{\frac{d^2x}{dt^2} = -\omega^2 x} \quad \text{III}$$

ANY EQUATION OF The form III has SOLUTION

$$\boxed{X(t) = A \cos \omega t} \quad \text{IV}$$

COMPARING II & III we conclude that for A spring

$$\omega^2 = \frac{k}{m}$$

$$\boxed{\omega = \sqrt{\frac{k}{m}}} \quad \text{V}$$

SO FOR A SPRING SOLUTION IV Becomes

$$\boxed{X(t) = A \cos \omega t \qquad \omega = \sqrt{\frac{k}{m}}} \quad \text{VI}$$

$A \equiv$ AMPLITUDE OF MOTION $\equiv$ MAXIMUM VALUE OF X

$\omega =$ Angular FREQUENCY

USEFUL RELATIONSHIPS:

$$\boxed{\begin{array}{ll} \omega T = 2\pi & T = \frac{1}{f} \\ \omega = 2\pi f & \end{array}}$$

$T \equiv$ PERIOD OF MOTION (s)

$f \equiv$ frequency of motion ($\frac{\text{cycles}}{s} = \text{Hz}$)

SHM

IN GENERAL, IF WE CAN APPLY NEWTON'S SECOND LAW AND OBTAIN AN EXPRESSION FOR THE acceleration OF THE form

$$a = -\frac{C}{m}x \qquad C \equiv \text{constant},$$

THE MOTION IS SHM with $\omega = \sqrt{\frac{C}{m}}$

FOR OUR SPRING THE COMPLETE DESCRIPTION OF THE MOTION IS:

$$\boxed{\begin{aligned} X(t) &= A\cos\omega t \\ v(t) &= -A\omega\sin\omega t \\ a(t) &= -A\omega^2\cos\omega t \\ \omega &= \sqrt{\frac{k}{m}} \end{aligned}}$$

VII

SHM FOR A MASS m ON A SPRING

The expressions FOR $v(t)$ and $a(t)$ can be OBTAINED BY PROJECTING UNIFORM CIRCULAR MOTION ONTO ONE DIMENSION USING A REFERENCE CIRCLE (see TEXTBOOK) OR BY TAKING DERIVITIVES WITH RESPECT TO TIME:

$$v(t) = \frac{dX(t)}{dt} \qquad a(t) = \frac{dv(t)}{dt}$$

Discussion #19

ELASTIC POTENTIAL ENERGY

LIKE GRAVITY, The restoring Force of a SPRING is conservative, And it is possible to define an ELASTIC POTENTIAL ENERGY. JUST AS FOR Gravity we CAN WRITE The WORK Done by the SPRING Force when stretching the spring From $X=0$ to $X=X$ as

$$W^{spring}_{0\to x} = EPE_0 - EPE_x$$

$$W^{spring}_{0\to x} = -(EPE_x - EPE_0) = -\Delta EPE$$

IF we DEFINE $EPE = 0$ when $X=0$ (SPRING is Unstretched)

$$W_{spring}(x) = -EPE(x)$$

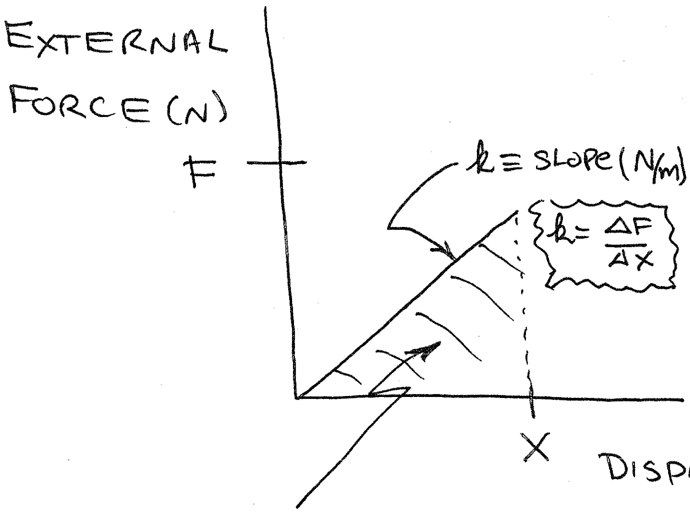
The SPRING does Negative WORK (opposes MOTION)

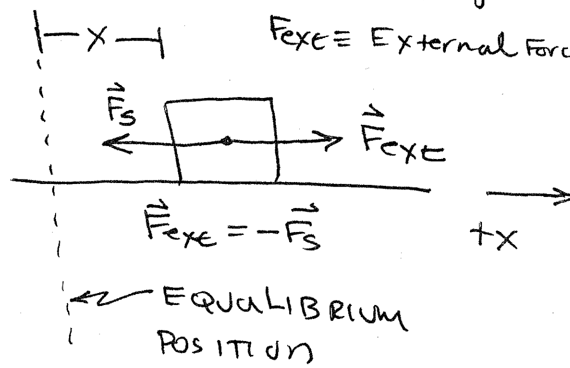
$\Rightarrow$ EXTERNAL FORCE DOES POSITIVE WORK AGAINST The SPRING FORCE AND THIS WORK IS The POSITIVE change IN EPE.

EPE

$F_s \equiv$ Spring Force

$F_{ext} \equiv$ External Force





Area under curve
= Work done by external
Force = ΔEPE

$$W_{ext} = \text{Area of triangle} = \frac{1}{2} F x$$

$$W_{ext} = \frac{1}{2} F x \qquad F = kx$$

$$W_{ext} = \Delta EPE = \frac{1}{2} k x^2$$

$$\boxed{EPE \equiv \frac{1}{2} k x^2 \qquad EPE \equiv 0 \text{ @ } x = 0}$$

WE CAN NOW UPDATE OUR WORK-ENERGY "BALANCE SHEET" TO INCLUDE ROTATIONAL KINETIC ENERGY AND ELASTIC POTENTIAL ENERGY:

EPE ≡ ELASTIC POTENTIAL ENERGY = $\frac{1}{2} k x^2$

GPE ≡ GRAVITATIONAL POTENTIAL ENERGY = mgh

TKE ≡ TRANSLATIONAL KINETIC ENERGY = $\frac{1}{2} m v^2$

RKE ≡ ROTATIONAL KINETIC ENERGY = $\frac{1}{2} I \omega^2$

W_{NC} ≡ WORK DONE BY NONCONSERVATIVE FORCES

$$\boxed{\begin{gathered} TKE_i + RKE_i + GPE_i + EPE_i + W_{NC} \\ \| \\ TKE_f + RKE_f + GPE_f + EPE_f \end{gathered}}$$

AS USUAL, WE SET ANY TERMS THAT DO NOT CONTRIBUTE TO ZERO