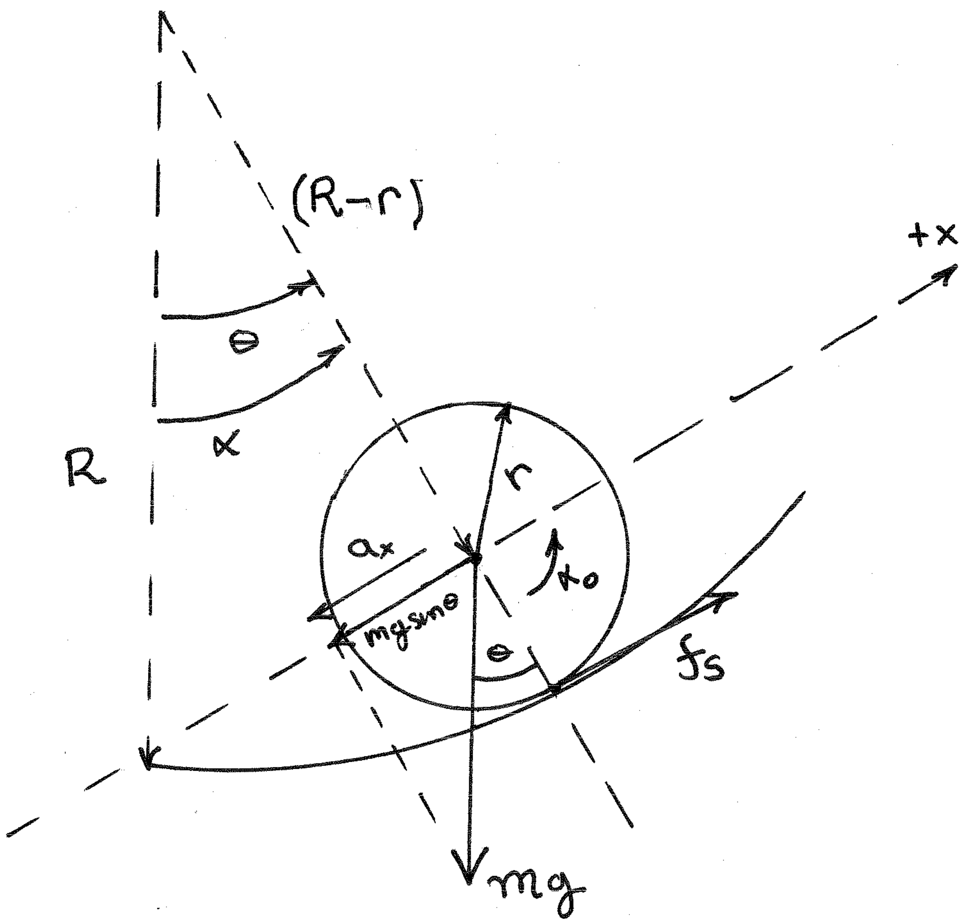
A ROUND OBJECT ROLLING IN A CYLINDRICAL DISH

A ROUND OBJECT OF RADIUS r ROLLS WITHOUT SLIPPING IN A BOWL WITH RADIUS R



AT 6-7-08

NEWTON'S SECOND LAW

$\sum F_x = m a_x$

USE MAGNITUDES AND EXPLICIT SIGNS

Ball Rolls in $-x$ $\rightarrow$ Restoring Force

$f_s - mg\sin\theta = -ma_x$

a_x is in $-x$ direction!

$$\boxed{m a_x = mg\sin\theta - f_s} \quad (\mathrm{I})$$

FOR Ball Rolling w/o SLIPPING $\Rightarrow$

$\alpha_o \equiv$ Angular acc. of object about it's center axis

Newton's second LAW: $\tau = I\alpha_o \Rightarrow$

$r f_s = I\alpha_o$ RWOS $\rightarrow$ $a_x = r\alpha_o \rightarrow \alpha_o = \frac{a_x}{r}$

$\Rightarrow r f_s = \frac{I a_x}{r}$ $\quad \boxed{f_s = \frac{I a_x}{r^2}} \ (\mathrm{II})$ $\quad \mathrm{II} \rightarrow \mathrm{I} \Rightarrow$

$$ma_x = mg\sin\theta - \frac{I a_x}{r^2}$$

$$ma_x + \frac{I a_x}{r^2} = mg\sin\theta$$

$$a_x\left[1 + \frac{I}{mr^2}\right] = g\sin\theta$$

$$\boxed{a_x = \frac{g\sin\theta}{\left[1 + \frac{I}{mr^2}\right]}} \quad (\mathrm{III})$$

$a_x = -(R-r)\,\alpha$

When α is ccw (+) a_x is in $-x$ direction

$$-(R-r)\alpha = \frac{g\sin\theta}{\left[1+\frac{I}{mr^2}\right]}$$

$$\boxed{\alpha = -\frac{g\sin\theta}{(R-r)\left[1+\frac{I}{mr^2}\right]}} \quad (\text{IV})$$

small $\theta \rightarrow$ $\sin\theta \approx \theta \rightarrow$

$$\boxed{\alpha \approx -\frac{g}{(R-r)\left[1+I/mr^2\right]}\,\theta} \quad (\text{V})$$

$$\boxed{\alpha \approx -\omega^2\theta}$$

V $\Rightarrow$ SHM with $\omega^2 = \dfrac{g}{(R-r)\left[1+I/mr^2\right]}$

$$\boxed{\omega = \sqrt{\frac{g}{(R-r)\left[1+I/mr^2\right]}} = \sqrt{\frac{g}{(R-r)\left[1+f_I\right]}}}$$

Note: $\omega = \dfrac{\omega_{pendulum}}{\sqrt{1+f_I}}$ $\quad \omega_{pendulum} = \sqrt{\dfrac{g}{(R-r)}}$

IF THE OBJECT WERE TO SLIDE w/o FRICTION IT would be IDENTICAL TO A simple PENDULUM OF LENGTH $(R-r)$ and

$\omega_{pendulum} = \sqrt{\frac{g}{(R-r)}}$. When The object Rolls IN THE BOWL The additional INERTIA DUE TO Rolling Reduces ω to:

$$\boxed{\omega_{rolling} = \frac{\omega_{pendulum}}{\left[1 + f_I\right]^{1/2}} \qquad f_I = \frac{I}{mr^2}}$$

IN BOTH Cases, The resulting ω does not depend on mass. TWO objects That can roll in a dish are a sphere with $f_I = \frac{2}{5}$ and a cylinder with $f_I = \frac{1}{2}$

$$\Rightarrow \omega_{\substack{\text{SOLID}\\\text{Sphere}\\\text{rolling}}} = \frac{\omega_{pendulum}}{\left[1 + \frac{2}{5}\right]^{1/2}} = \sqrt{\frac{5}{7}}\,\omega_{pendulum}$$

$$\omega_{\substack{\text{SOLID}\\\text{cylinder}\\\text{rolling}}} = \frac{\omega_{pendulum}}{\left[1 + 1/2\right]^{1/2}} = \sqrt{\frac{2}{3}}\,\omega_{pendulum}$$