EXAMPLE 12

Rolling cylinders

WORK ENERGY THEOREM

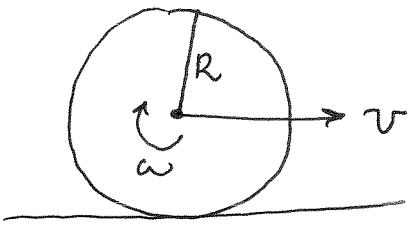
$$\cancel{TKE_i}^{\,0} + \cancel{RKE_i}^{\,0} + PE_i + \cancel{W_{NC}}^{\,0} = TKE_f + RKE_f + \cancel{PE_f}^{\,0}$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + 0$$

$$\boxed{mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2}$$

Rolling w/o SLIPPING =>

$$\boxed{v = R\omega} \qquad \boxed{\omega = \frac{v}{R}}$$



=>

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\left[\frac{v}{R}\right]^2$$

$$2gh = v^2 + \frac{I}{m}\frac{v^2}{R^2} = v^2\left[1 + \frac{I}{mR^2}\right]$$

$$v = \sqrt{\frac{2gh}{1 + I/mR^2}}$$

$$v = \sqrt{\frac{2gh}{\left[1 + \frac{I}{MR^2}\right]}}$$

Compare a Ring to a SOLID CYLINDER

Ring: $I_{Ring} = MR^2$

Cylinder $= \frac{1}{2} I_{Ring} = \frac{1}{2} MR^2$

OBJECT	$[1 + I/MR^2]$	$\frac{1}{[1 + I/MR^2]}$	
Ring	2	$\frac{1}{2}$	0.5
Cylinder	$1 + \frac{1}{2} = \frac{3}{2}$	$\frac{2}{3}$	0.67
SOLID SPHERE	$1 + \frac{2}{5} = \frac{7}{5}$	$5/7$	0.714
Hollow SPHERE	$1 + \frac{2}{3} = \frac{5}{3}$	$3/5$	0.6

RANK Shapes from Largest v to Smallest

SOLID SPHERE > CYLINDER > Hollow SPHERE > Ring