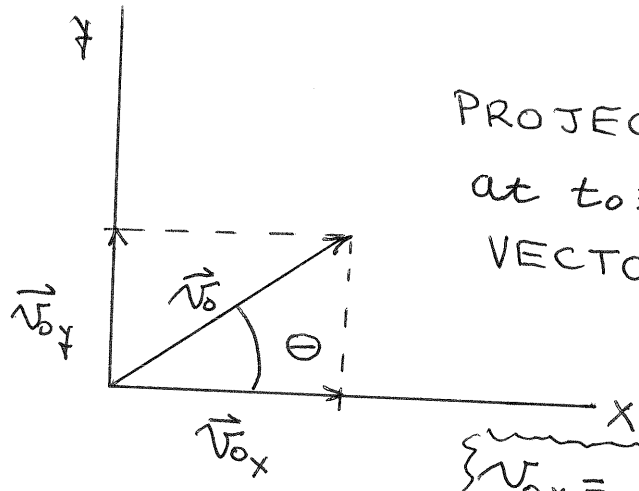
PROJECTILES

X AND Y COMPONENTS OF MOTION ARE INDEPENDENT OF EACH OTHER

SOLUTION TO TWO INDEPENDENT 1D MOTIONS (USING KINEMATICS) WHICH OCCUR SIMULTANEOUSLY



PROJECTILE IS LAUNCHED at $t_0 \equiv 0$ WITH INITIAL VELOCITY VECTOR $\vec{v}_0$

$$v_{0x} = |\vec{v}_0| \cos\theta = v_0 \cos\theta$$

$$v_{0y} = |\vec{v}_0| \sin\theta = v_0 \sin\theta$$

X MOTION

THERE IS NO ACCELERATION IN THE X DIRECTION SO THE X VELOCITY NEVER CHANGES

$v_{0x} \equiv$ CONSTANT X VELOCITY

$v_{0x} = v_0 \cos\theta$

$$x(t) = [v_0 \cos\theta] \cdot t$$

y MOTION

THE y MOTION FOR A PROJECTILE IS SIMPLY A 1D "FREE FALL" WITH $\vec{g}$ ACTING DOWNWARD AND THE PROJECTILE LAUNCHED UPWARD WITH INITIAL y VELOCITY $v_{oy} = v_o \sin\theta$

CHOOSE UPWARD $\equiv$ +y DIRECTION $\Rightarrow$

$v_{oy} = +v_o \sin\theta$ $\qquad$ $y_o \equiv 0$ (call ground $y=0$)

$a = -g$

$$v_y = v_{oy} - gt \qquad y(t) = v_{oy} t - \frac{1}{2} g t^2$$

$$v_y(t) = v_o \sin\theta - gt$$

$$y(t) = [v_o \sin\theta]\, t - \frac{1}{2} g t^2$$

ALSO $\quad v_y^2(y) = v_{oy}^2 - 2gy \quad \Rightarrow$

HEIGHT H

AT MAXIMUM HEIGHT H $v_y = 0$

$$0 = v_{oy}^2 - 2gH \qquad H = \frac{v_{oy}^2}{2g} = \frac{v_o^2 \sin^2\theta}{2g}$$

RANGE R

$R = v_{0x}\, t_f$ $\quad t_f \equiv$ "TIME OF FLIGHT"

$$y(t) = \cancel{y_0}^{\,0} + v_{0y} t - \frac{1}{2} g t^2$$

WHEN $t = t_f$ $\quad y(t_f) = 0$ (PROJECTILE HAS RETURNED TO GROUND $\equiv y = 0$)

$$\Rightarrow \quad y(t_f) = 0 = v_{0y} t_f - \frac{1}{2} g t_f^2$$

$$\Rightarrow \quad t_f \left(v_{0y} - \frac{1}{2} g t_f \right) = 0$$

$$t_f = 0 \quad \text{OR} \quad t_f = \frac{2 v_{0y}}{g} \qquad v_{0y} = v_0 \sin\theta$$

$$\boxed{t_f = \frac{2 v_0 \sin\theta}{g}}$$

$$R = v_{0x}\, t_f = [v_0 \cos\theta]\, t_f = [v_0 \cos\theta] \left[\frac{2 v_0 \sin\theta}{g} \right]$$

$$R = \frac{2 v_0^2 \sin\theta \cos\theta}{g} \qquad \boxed{2 \sin\theta\cos\theta = \sin 2\theta}$$

$$\boxed{R = \frac{v_0^2 \sin 2\theta}{g}}$$