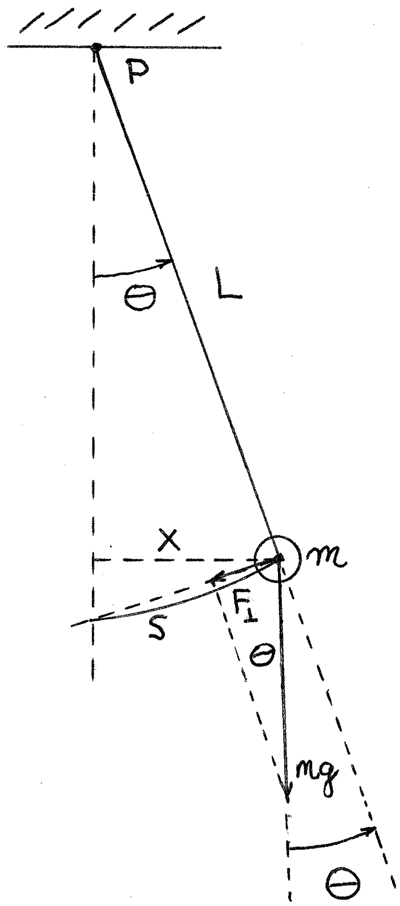
THE SIMPLE PENDULUM

DISCUSSION #20



THE TORQUE DUE TO THE WEIGHT OF THE PENDULUM BOB ABOUT AN AXIS THROUGH THE PIVOT POINT P IS

$$\tau_P = -F_\perp L \qquad F_\perp = mg\sin\theta$$

$$\tau_P = -mg\sin\theta L \qquad \sin\theta = \frac{X}{L}$$

$$\boxed{\tau_P = -mgX}$$

τ_p is in the opposite direction (cw) to that of θ (ccw) $\Rightarrow$ $\tau_p = -mgx$ is a restoring torque.

Now make the approximation that for small angles θ the distance x is approximately the arc length s:

$$x \approx s = L\theta$$

$$\tau_p = -mgx \approx -mgL\theta$$

$$\boxed{\tau_p \approx -mgL\theta} \quad \text{small } \theta$$

Now apply Newton's second law for the torque τ_p:

$$\tau_p = I\alpha = -mgL\theta$$

$$\boxed{\alpha \approx -\frac{mgL\theta}{I}} \quad \text{small } \theta$$

The moment of inertia for the pendulum bob about the axis P is

$$\boxed{I = mL^2}$$

SO WE CAN WRITE THE ANGULAR ACCELERATION AS:

$$\alpha \approx -\frac{mgL}{mL^2}\theta$$

$$\alpha \approx -\frac{g}{L}\theta$$ SMALL θ

RECALL FOR THE MASS/SPRING SYSTEM NEWTON'S SECOND LAW GAVE

$$a = -\frac{k}{m}x$$

WE RECOGNIZE THAT THIS EQUATION HAS A SOLUTION $x(t) = A\cos(\omega t)$ WITH $\omega = \sqrt{\frac{k}{m}}$ WHICH IS SIMPLE HARMONIC MOTION (SHM).

WE NOW HAVE $\alpha = -\frac{g}{L}\theta$ AND AGAIN THIS HAS THE SAME MATHEMATICAL SOLUTION WITH $\omega = \sqrt{\frac{g}{L}}$ SIMPLE PENDULUM, SMALL θ

SO FOR SMALL ANGLES θ, THE ANGLE OF A SIMPLE PENDULUM EXECUTES SHM:

$$\theta(t) \approx A_\theta \cos(\omega t)$$

$$\omega = \sqrt{\frac{g}{L}} \qquad \text{small } \theta$$

$A_\theta \equiv$ AMPLITUDE $\equiv$ MAXIMUM ANGLE OF MOTION (IN RADIANS) $\equiv$ ANGLE AT RELEASE FROM REST

THE REMARKABLE RESULT IS THAT THE ANGULAR FREQUENCY ω depends only on g AND L FOR THE PENDULUM, NOT THE MASS OF THE BOB.

THE PHYSICAL PENDULUM

FOR THE SIMPLE PENDULUM WE FOUND

$$\alpha = -\frac{mgL}{I}\theta$$

$$\alpha \equiv -\omega^2\theta$$

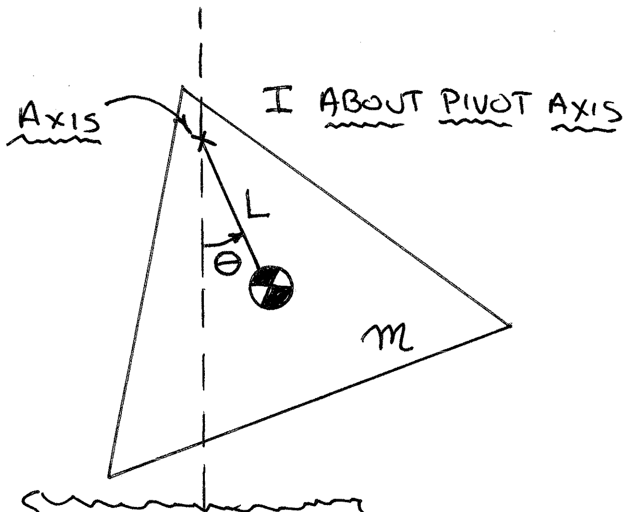
$$\omega = \sqrt{\frac{mgL}{I}}$$

SUBSTITUTING THE PARTICULAR MOMENT OF INERTIA WE OBTAINED $\omega = \sqrt{\frac{g}{L}}$ for A SIMPLE PENDULUM. THE ABOVE EXPRESSIONS CAN BE GENERALIZED TO EXTENDED OBJECTS which ARE PIVOTED ABOUT AN AXIS and SET INTO OSCILLATION AT SMALL ANGLES.

THIS IS CALLED A PHYSICAL PENDULUM.

THE DISTANCE L IS NOW THE DISTANCE FROM THE PIVOT AXIS TO THE OBJECT'S CENTER OF GRAVITY. I IS THE OBJECT'S

MOMENT OF INERTIA ABOUT THE PIVOT AXIS:



$$\omega = \sqrt{\frac{mgL}{I}}$$

PHYSICAL PENDULUM

Small θ

By MEASURING ω YOU CAN ACCURATELY DETERMINE I EXPERIMENTALLY.