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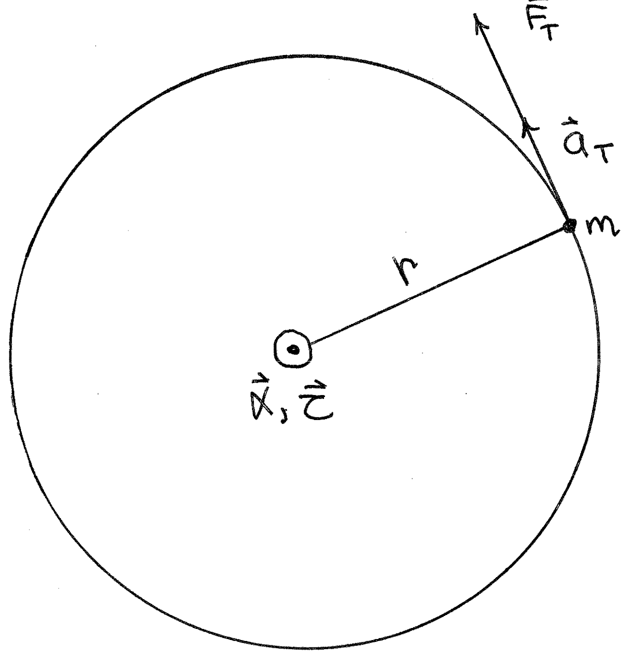
Newton's Second Law for Rotational Motion



$$\vec{F}_T = m\vec{a}_T$$

$$|\vec{\tau}| = |\vec{F}_T| \cdot r = m|\vec{a}_T| \cdot r$$

$$|\vec{a}_T| = r|\vec{\alpha}|$$

$$|\vec{\tau}| = mr(r|\vec{\alpha}|)$$

$$|\vec{\tau}| = mr^2|\vec{\alpha}|$$

$$\boxed{\vec{\tau} = \underbrace{[mr^2]}_{I}\vec{\alpha}}$$

$$\vec{F} = m\vec{a} \iff \vec{\tau} = I\vec{\alpha} \quad \text{(RESTRICTED)}$$

$$\vec{F} = \frac{\Delta\vec{p}}{\Delta t} \iff \vec{\tau} = \frac{\Delta\vec{L}}{\Delta t} \quad \text{(General)}$$

$\vec{L} \equiv$ Angular Momentum => More to come!

$mr^2 \equiv$ Moment of INERTIA for Particle of mass m about ROTATION AXIS => PLAYS The SAME Role as Mass does FOR Linear motion in Newton's second Law.

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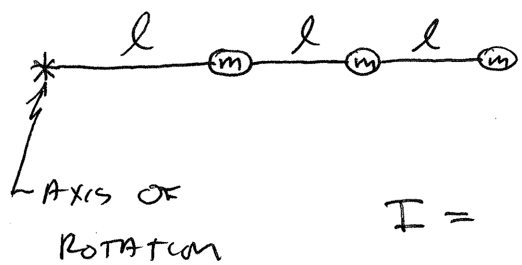
IF we have multiple masses at different distances from the rotation Axis, we add up The Moments of INERTIA FOR the Individual masses ⇒

$$I_{total} = \sum_{i=1}^{N} m_i r_i$$

N PARTICLES each with mass m_i at distance r_i from rotation axis.

EXAMPLE: Beads on a wire



$$I = m\ell^2 + m(2\ell)^2 + m(3\ell)^2$$

$$I = m\ell^2(1 + 4 + 9)$$

$$I = 14m\ell^2$$

FOR EXTENded Objects we use the methods of CALCULUS to break up The object into small mass elements and add up the Contribution of each.

We will QUOTE The results without doing the Calculations...

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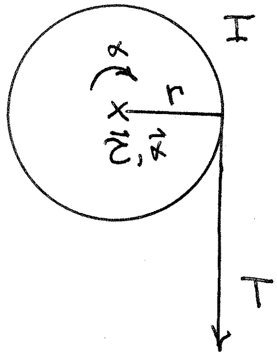
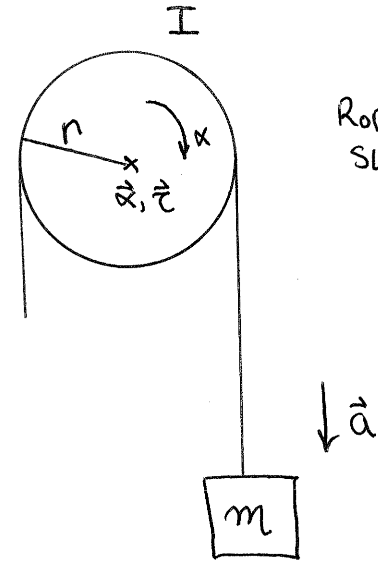
EXAMPLE: A MASSIVE PULLEY





Rope does not SLIP => $a = r\alpha$

Newton's 2nd Law:

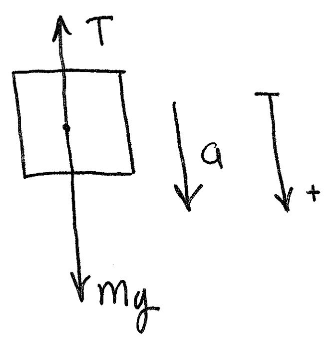
$$\vec{\tau} = I\vec{\alpha}$$

$$\tau = rT$$

$$\alpha = \frac{a}{r}$$

$$rT = I\left(\frac{a}{r}\right)$$

$$\boxed{T = I\frac{a}{r^2}} \quad (I)$$



Newton's second Law:

$$mg - T = ma$$

$$\boxed{T = m(g-a)} \quad (II)$$

Find a =>

$$I = II \qquad \frac{Ia}{r^2} = m(g-a)$$

$$a\left(\frac{I}{r^2}\right) = mg - ma$$

$$a\left(m + \frac{I}{r^2}\right) = mg$$

$$a = \frac{mg}{\left(m + \frac{I}{r^2}\right)}$$

$$\boxed{a = \frac{g}{1 + \frac{I}{mr^2}}}$$

$$\boxed{a = \frac{g}{\left[1 + \frac{I}{mr^2}\right]}}$$

N.B. => $I \to 0$ => $a \to g$ ✓