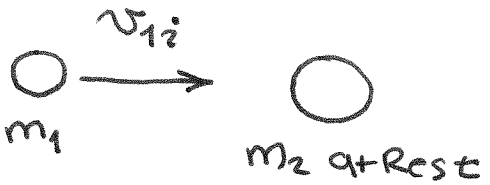
A PERFECTLY ELASTIC 1-D COLLISION

PHYSICS 125
LECTURE 13
AT 10/15/08



BEFORE

AFTER

CONSERVE MOMENTUM

$$\text{I.} \quad m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f}$$

CONSERVE KINETIC ENERGY ($\equiv$ ELASTIC COLLISION)

$$\text{II} \quad \frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

SOLVE I for v_{2f} $\longrightarrow$

$$m_1 v_{1i} - m_1 v_{1f} = m_2 v_{2f}$$

$$v_{2f} = \frac{m_1}{m_2} \left[v_{1i} - v_{1f} \right] \longrightarrow \text{II}$$

$$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 \left[\frac{m_1}{m_2} (v_{1i} - v_{1f}) \right]^2$$

$$m_1 v_{1i}^2 = m_1 v_{1f}^2 + m_2 \left[\frac{m_1}{m_2} \right]^2 \left[v_{1i}^2 - 2 v_{1i} v_{1f} + v_{1f}^2 \right]$$

$$v_{1i}^2 = v_{1f}^2 + \frac{m_2}{m_1} \left[\frac{m_1}{m_2} \right]^2 \left[v_{1i}^2 - 2 v_{1i} v_{1f} + v_{1f}^2 \right]$$

$$1 = \left[\frac{v_{1f}}{v_{1i}} \right]^2 + \frac{m_1}{m_2} \left[1 - 2 \frac{v_{1f}}{v_{1i}} + \left[\frac{v_{1f}}{v_{1i}} \right]^2 \right]$$

$$x \equiv \frac{v_{1f}}{v_{1i}} \longrightarrow$$

$$1 = x^2 + \frac{m_1}{m_2} - 2 \frac{m_1}{m_2} x + \frac{m_1}{m_2} x^2$$

$$1 = x^2 \left[1 + \frac{m_1}{m_2} \right] - 2 \frac{m_1}{m_2} x + \frac{m_1}{m_2}$$

$$x^2 \left[1 + \frac{m_1}{m_2} \right] - 2 \frac{m_1}{m_2} x + \left[\frac{m_1}{m_2} - 1 \right] = 0$$

$$x^2 \left[\frac{m_2 + m_1}{m_2} \right] - 2 \frac{m_1}{m_2} x + \left[\frac{m_1 - m_2}{m_2} \right] = 0$$

SOLVE QUADRATIC FOR $x = \frac{v_{1f}}{v_{1i}}$

$$x = \frac{2\frac{m_1}{m_2} \pm \left[4\left[\frac{m_1}{m_2}\right]^2 - 4\left[\frac{m_1+m_2}{m_2}\right]\left[\frac{m_1-m_2}{m_2}\right]\right]^{1/2}}{2\left[\frac{m_2+m_1}{m_2}\right]}$$

$$x = \frac{\cancel{2}\frac{m_1}{m_2} \pm \frac{\cancel{2}}{m_2}\left[m_1^2 - [m_1+m_2][m_1-m_2]\right]^{1/2}}{\frac{\cancel{2}}{m_2}[m_2+m_1]}$$

$$x = \frac{m_1 \pm \left[m_1^2 - \{m_1^2 - m_2^2\}\right]^{1/2}}{[m_1+m_2]}$$

$$x = \frac{m_1 \pm [m_2^2]^{1/2}}{[m_1+m_2]}$$

$$\boxed{x = \frac{m_1 \pm m_2}{m_1+m_2}}$$

THE TWO ROOTS ARE:

$$\chi_+ = \frac{m_1 + m_2}{m_1 + m_2} = 1 \quad \text{FOR ALL } m_1 \text{ \& } m_2$$

$\Rightarrow$ UNPHYSICAL

$$\chi_- = \frac{m_1 - m_2}{m_1 + m_2} \Longrightarrow$$

$$\frac{v_{1f}}{v_{1i}} = \frac{m_1 - m_2}{m_1 + m_2}$$

$$\boxed{v_{1f} = \left[\frac{m_1 - m_2}{m_1 + m_2}\right] v_{1i}}$$

NOW SUBSTITUTE v_{1f} INTO I

AND SOLVE FOR $v_{2f} \Longrightarrow$

$$v_{2f} = \frac{m_1}{m_2}\left[v_{1i} - v_{1f}\right] \quad \text{(FROM I)}$$

$$v_{2f} = \frac{m_1}{m_2}\left[v_{1i} - \left[\frac{m_1 - m_2}{m_1 + m_2}\right] v_{1i}\right]$$

$$v_{2f} = \frac{m_1}{m_2}\left[1 - \left[\frac{m_1 - m_2}{m_1 + m_2}\right]\right] v_{1i}$$

$$v_{2f} = \frac{m_1}{m_2}\left[\frac{(m_1 + m_2) - (m_1 - m_2)}{m_1 + m_2}\right] v_{1i}$$

$$v_{2f} = \frac{m_1}{m_2}\left[\frac{2\,m_2}{m_1 + m_2}\right] v_{1i}$$

$$\boxed{v_{2f} = \left[\frac{2\,m_1}{m_1 + m_2}\right] v_{1i}}$$

AND

$$\boxed{v_{1f} = \left[\frac{m_1 - m_2}{m_1 + m_2}\right] v_{1i}}$$

FOR A PERFECTLY ELASTIC 1-D COLLISION WITH m_2 INITIALLY AT REST.