Displacement, Velocity and Acceleration

To describe motion we need to be able to describe an object's position, displacement (change in position), velocity (rate of change of position) and acceleration (rate of change of velocity). Here's how these quantities are defined:

$\vec{x} \equiv$ Position Vector

$\vec{d} \equiv$ Displacement Vector $\equiv$ Change in Position Vector

$\vec{d} \equiv \Delta\vec{x} = \vec{x} - \vec{x}_{initial} \equiv \vec{x} - \vec{x}_0$ If $\vec{x}_0 = 0$ (motion begins at origin) then $\vec{d} = \vec{x}$

$\Delta t \equiv$ Time interval $= t - t_{initial} \equiv t - t_0$ If $t_0 = 0$ (clock is started at $t = 0$) then $\Delta t = t$

$\bar{\vec{v}} \equiv$ Average Velocity (Vector) $\equiv$ Average Change in Position With Time

$\bar{\vec{v}} \equiv \frac{\Delta\vec{x}}{\Delta t} = \frac{\vec{x} - \vec{x}_0}{t - t_0}$ If $\vec{x}_0 = 0$ and $t_0 = 0$ then $\bar{\vec{v}} = \frac{\vec{x}}{t}$

$\bar{v} \equiv$ Average *Speed* (Scalar) $\equiv \left|\bar{\vec{v}}\right| = \frac{\left|\Delta\vec{x}\right|}{\Delta t}$

$\bar{\vec{a}} \equiv$ Average Acceleration (Vector) $\equiv$ Average Change in Velocity With time

$\bar{\vec{a}} \equiv \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v} - \vec{v}_{initial}}{t - t_{initial}} \equiv \frac{\vec{v} - \vec{v}_0}{t - t_0}$ If $\vec{v}_0 = 0$ and $t_0 = 0$ then $\bar{\vec{a}} = \frac{\vec{v}}{t}$

The definitions given above are average values defined over a finite interval of time Δt. The capitol Greek symbol Delta ("Δ") is used to mean "change in a quantity". The horizontal bar over the symbol for a quantity (e.g. $\bar{\vec{a}}$) means "average" for both vectors and scalars.

Although we will not *use* the methods of Calculus as a mathematical tool in our course, it is nonetheless *conceptually* helpful to understand how the *instantaneous* values of velocity and acceleration relate to the definitions given above for the *average* values by examining the limit as the time interval Δt approaches zero:

$$\vec{v} \equiv \text{ Instantaneous Velocity} \equiv \lim_{\Delta t \to 0} \frac{\Delta \vec{x}}{\Delta t} \equiv \frac{d\vec{x}}{dt}$$

$$\vec{a} \equiv \text{ Instantaneous Acceleration} \equiv \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} \equiv \frac{d\vec{v}}{dt} = \frac{d^2\vec{x}}{dt^2}$$

As we consider smaller and smaller time intervals, the *average* values for velocity and acceleration approach the *instantaneous* values defined at a single point, and the instantaneous velocity and acceleration correspond to *time derivatives* of position and velocity respectively. Further, the acceleration is the *second derivative* of the position with respect to time.

Note that since position, velocity and acceleration are *vector* quantities, they can change in both *magnitude* and *direction*. For a vector quantity to be constant it must therefore constant in *both* magnitude *and* direction. Thus an object moving in a straight line with a constant *speed* (remember that the speed is the *magnitude* of the velocity) has a constant velocity vector, but an object moving in a *circle* with a constant speed does not, since the *direction* of the velocity vector is constantly changing even though the *speed* remains constant.

For kinematics problems in one dimension, we customarily drop the explicit vector arrows above the position, velocity and acceleration vectors and understand that the algebraic minus signs take care of the vector nature of these quantities.

Conceptually a *derivative* represents the *slope* of a graph. For one-dimensional kinematics problems the instantaneous *velocity* (with algebraic sign, still a *vector* in one dimension) is the *slope of the position vs. time graph at any point on the graph.* A *constant velocity* therefore corresponds to a *straight line* in the x vs. t graph. Similarly, the *instantaneous acceleration* at any point is the *slope* of the *velocity vs. time graph* and a *straight line* segment in the v vs. t graph corresponds to an interval of *constant acceleration*. It is important that you be able to deduce velocity and acceleration values from graphs of x vs. t and v vs. t respectively and we will practice this in our problems.