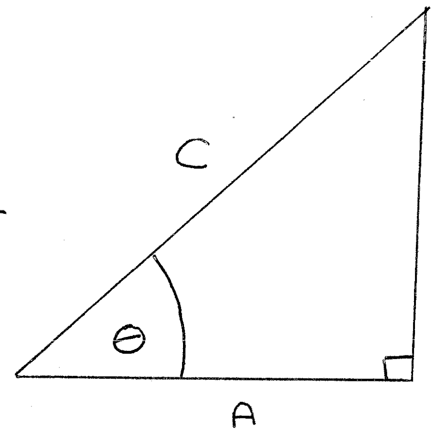
DISCUSSION Ø

# MATHEMATICAL TOOLS

## TRIG FUNCTIONS




PYTHAGOREAN THEOREM:

$$\boxed{C^2 = A^2 + B^2}$$

$$\sin\theta = \frac{\text{OPPOSITE}}{\text{hYPOTENUSE}} = \frac{B}{C}$$

$$\cos\theta = \frac{\text{ADJACENT}}{\text{hYPOTENUSE}} = \frac{A}{C}$$

$$\tan\theta = \frac{\text{OPPOSITE}}{\text{ADJACENT}} = \frac{B}{A}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{C}{B} \qquad \sec\theta = \frac{1}{\cos\theta} = \frac{C}{A}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{A}{B}$$

## USEFUL IDENTITIES

$$\sin^2\theta + \cos^2\theta = 1 \qquad \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{B}{C}\cdot\frac{C}{A} = \frac{B}{A} \checkmark$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{A}{C}\cdot\frac{C}{B} = \frac{A}{B} \checkmark$$

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# QUADRATIC FORMULA

THE QUADRATIC EQUATION:

$$\boxed{ax^2 + bx + c = 0}$$

a, B, C ≡ constants

HAS ROOTS:

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

# SCALING

OFTEN WE WANT TO KNOW HOW A QUANTITY CHANGES WHEN ANOTHER QUANTITY UPON WHICH IT DEPENDS IS CHANGED.

## EXAMPLE

HOW DOES THE AREA OF A CIRCLE CHANGE WHEN ITS CIRCUMFERENCE IS DOUBLED?

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SOLUTION

$\boxed{\text{I. } A = \pi R^2}$ $\boxed{\text{II. } C = 2\pi R}$

R ≡ RADIUS

C ≡ CIRCUMFERENCE

A ≡ AREA

FIRST DERIVE A RELATIONSHIP BETWEEN AREA AND CIRCUMFERENCE BY ELIMINATING THE RADIUS BETWEEN EQUATIONS I & II

$$\text{II} \quad C = 2\pi R \implies \boxed{R = \frac{C}{2\pi}} \implies \text{I}$$

$$\text{I.} \quad A = \pi R^2 = \pi \left[\frac{C}{2\pi}\right]^2 = \frac{\pi C^2}{4\pi^2} = \frac{C^2}{4\pi}.$$

$$\boxed{A = \frac{C^2}{4\pi}}$$

NOW COMPARE THE AREA OF TWO CIRCLES BY TAKING THEIR RATIO

$$A_1 = \frac{C_1^2}{4\pi} \quad A_2 = \frac{C_2^2}{4\pi} \quad \frac{A_2}{A_1} = \frac{C_2^2}{4\pi} \cdot \frac{4\pi}{C_1^2}$$

$$\boxed{\frac{A_2}{A_1} = \frac{C_2^2}{C_1^2}}$$

"Double the Circumference"

$\implies$ $C_2 = 2C_1$

$$\frac{A_2}{A_1} = \frac{[2C_1]^2}{C_1^2} = \frac{4\,C_1^2}{C_1^2} = 4$$

$\boxed{A_2 = 4A_1}$

AREA INCREASES by FACTOR OF 4