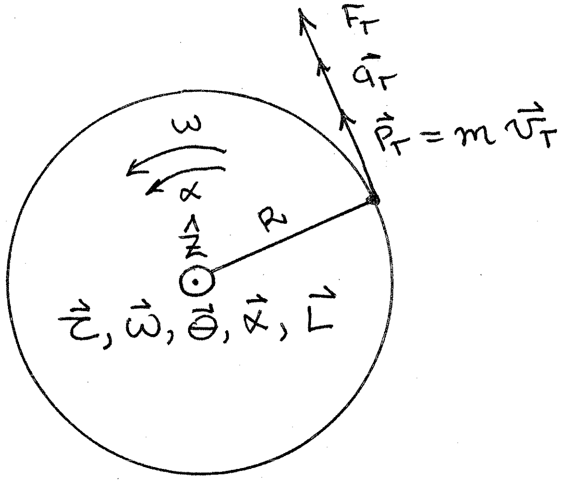
NEWTON'S SECOND LAW AND ANGULAR MOMENTUM



$\hat{z}$ ≡ UNIT VECTOR OUT OF PAGE

$\vec{\tau} \equiv \tau \hat{z}$

$\vec{\omega} \equiv \omega \hat{z}$

$\vec{\theta} \equiv \theta \hat{z}$

$\vec{\alpha} \equiv \alpha \hat{z}$

$\vec{L} \equiv L \hat{z}$

ALL POINT OUT OF PAGE AS VECTORS

ANGULAR MOMENTUM

$$\vec{L} \equiv R P_T \hat{z}$$

$$L = R P_T = R(m v_T) = Rm(R\omega)$$

$$L = mR^2\omega$$

$$L = I\omega$$

$$\vec{L} = I\vec{\omega}$$

NEWTON'S SECOND LAW FOR ROTATION

$$\vec{\tau} = \frac{\Delta \vec{L}}{\Delta t}$$

ALWAYS TRUE, WITHOUT RESTRICTIONS

PROOF:

$$\tau = R F_T = R(m a_T) = \frac{R(m v_{T_f} - m v_{T_i})}{\Delta t}$$

$$\tau = \frac{R(P_{Tf} - P_{Ti})}{\Delta t} = \frac{[RP_T]_f - [RP_T]_i}{\Delta t} = \frac{\Delta L}{\Delta t}$$

11/14/06

SO $\tau = \dfrac{\Delta L}{\Delta t}$ $\quad \vec{\tau} = \tau \hat{z} \quad \vec{L} = L\hat{z} \Rightarrow$

$$\boxed{\vec{\tau} = \frac{\Delta \vec{L}}{\Delta t}}$$

Q.E.D.

IF m & R ARE HELD CONSTANT $\Longrightarrow$

$$\tau = \frac{R P_{T_f} - R P_{T_i}}{\Delta t} = \frac{R(m v_{T_f} - m v_{T_i})}{\Delta t}; \quad v_T = R\omega \Rightarrow$$

$$\tau = \frac{mR(R\omega_f - R\omega_i)}{\Delta t} = mR^2 \frac{(\omega_f - \omega_i)}{\Delta t} = mR^2 \alpha$$

$$\boxed{\begin{array}{c} \tau = I\alpha \\ \vec{\tau} = I\vec{\alpha} \end{array}} \qquad \boxed{I \equiv mR^2}$$

RESTRICTED: ONLY TRUE FOR m & R CONSTANT $\Rightarrow$

ONLY TRUE FOR $I \equiv$ CONSTANT

Nov 14, 2006

MOMENTUM CONSERVATION

NEWTON'S SECOND LAW

GENERAL W/O RESTRICTIONS

LINEAR	ANGULAR
$\vec{F} = \frac{\Delta \vec{P}}{\Delta t}$	$\vec{\tau} = \frac{\Delta \vec{L}}{\Delta t}$
$\Downarrow$	$\Downarrow$
$\vec{F} = m\vec{a}$	$\vec{\tau} = I\vec{\alpha}$
RESTRICTED TO $m =$ CONSTANT	RESTRICTED TO $m =$ CONSTANT, $R =$ CONSTANT $\Rightarrow I \equiv$ CONSTANT

MOMENTUM CONSERVATION

LINEAR	ANGULAR
$\vec{F} = 0 \Rightarrow \frac{\Delta \vec{P}}{\Delta t} = 0$	$\vec{\tau} = 0 \Longrightarrow \frac{\Delta \vec{L}}{\Delta t} = 0$
$\Rightarrow \vec{P}$ does NOT change w/time	$\Longrightarrow \vec{L}$ does not change w/time
$\Rightarrow$ $\vec{P}$ IS CONSERVED	$\Longrightarrow$ $\vec{L}$ IS CONSERVED

IMPULSE – MOMENTUM

NOVEMBER 14, 2006

LINEAR

$$\vec{F} = \frac{\Delta \vec{P}}{\Delta t}$$

$$\Longrightarrow \quad \Delta \vec{P} = \vec{F} \Delta t$$

"Change in momentum = Force x time"

ANGULAR

$$\vec{\tau} = \frac{\Delta \vec{L}}{\Delta t}$$

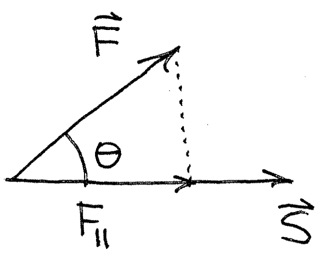
$$\Delta \vec{L} = \vec{\tau} \Delta t$$

"Change in Angular Momentum = torque x time"

WORK – ENERGY THEOREM

WORK

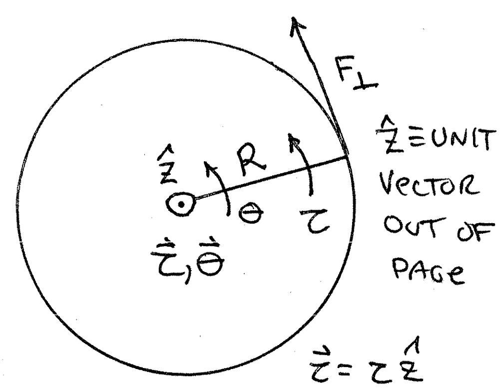
$$W_{\vec{F}} = F_{\parallel} \cdot S = F S \cos\theta$$



$W_{\vec{F}}$ is Negative if $\cos\theta$ is negative $(90° < \theta < 270°)$

$$W_{\vec{\tau}} = \tau \theta$$

$W_{\vec{\tau}}$ is Negative if $\vec{\theta}$ and $\vec{\tau}$ are in Opposite Directions (one cw, the other ccw)



$$\vec{\tau} = \tau \hat{z}$$

$$\vec{\theta} = \theta \hat{z}$$

$$\tau = R F_{\perp}$$

τ ccw $\equiv (+)$ (out of page BY RHR)

θ ccw $\equiv (+)$

WORK ENERGY THEOREM

$$W_{\vec{F}_{net}} = \Delta KE = KE_f - KE_i$$

$$W_{\vec{F}_{net}} = \frac{1}{2} m_f v_f^2 - \frac{1}{2} m_i v_i^2$$

$$W_{\vec{F}_{net}} = \frac{P_f^2}{2m_f} - \frac{P_i^2}{2m_i}$$

"WORK DONE BY NET EXTERNAL FORCE EQUALS CHANGE IN LINEAR KINETIC ENERGY"

$$W_{\vec{\tau}_{net}} = \Delta KE_{ROT}$$

$$W_{\vec{\tau}_{net}} = \frac{1}{2} I_f \omega_f^2 - \frac{1}{2} I_i \omega_i^2$$

$$W_{\vec{\tau}_{net}} = \frac{L_f^2}{2I_f} - \frac{L_i^2}{I_i}$$

"WORK DONE BY NET EXTERNAL TORQUE EQUALS CHANGE IN ROTATIONAL KINETIC ENERGY"